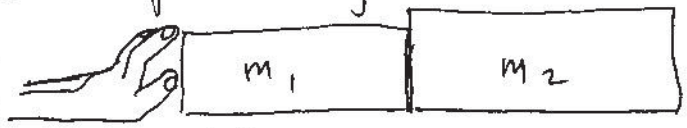
In the lecture, I will cover examples that are marked with large dots. You must be able to do all these examples! Questions about any of these examples? Please post them on WebCT, so that we can discuss them.

• Ex 4.4 of text

Pushing books
No friction

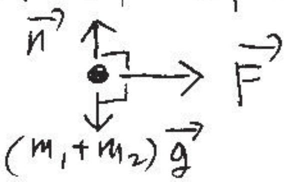
["Compound object" and the 3rd law]



$\rightarrow \vec{F}$ by hand

Question: Force exerted by m_2 on m_1 ?

① F. b. d. of $m_1 + m_2$?



"Compound object"

No vertical motion

$$\Rightarrow \vec{n} + (m_1+m_2)\vec{g} = 0$$

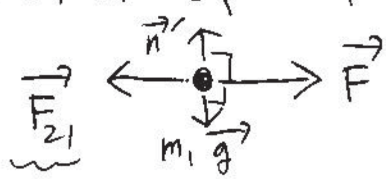
Horizontal motion

$$\Rightarrow \vec{F} = m\vec{a}$$

$$m = m_1 + m_2$$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m_1+m_2} \quad \ldots (i)$$

② F. b. d. of m_1 ?



by 2 on 1

No vertical motion

$$\Rightarrow \vec{n}' + m_1\vec{g} = 0$$

Horizontal motion

$$\vec{F} + \vec{F}_{21} = m_1\vec{a}$$

$$\therefore \vec{F}_{21} = m_1\vec{a} - \vec{F}$$

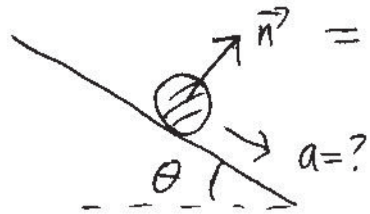
$$= \frac{m_1\vec{F}}{m_1+m_2} - \vec{F} = \frac{-m_2}{m_1+m_2}\vec{F}$$

use (i)

This is the answer.

Note: the vertical motion or the lack thereof was considered only for completeness. It does not contribute to the answer.

Ex 5.1 of text   Skier on a downward slope



$\vec{n}$: force exerted by snow on the skier

a : skier's acceleration

θ : $32°$, $m = 65\ kg$

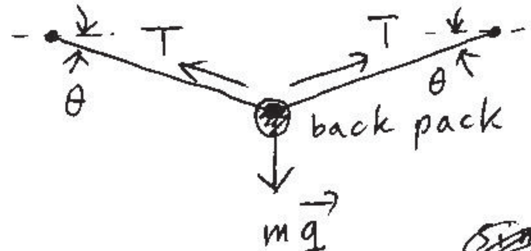
From ~~#~~ exam prob 1 (Sec. 1)

$\Rightarrow$ $a = g \sin\theta$

$n = mg \cos\theta$

Ex 5.2 of text Tension ?

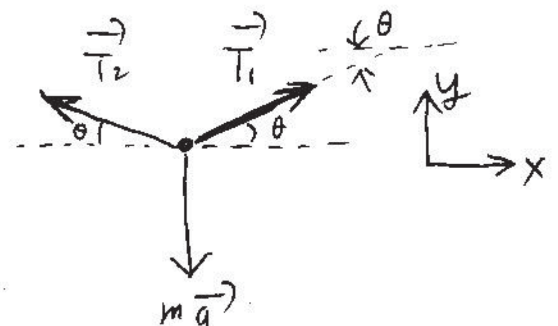
"bear precautions"



$T = ?$

(Symmetric shape) The tension must be the same on the two ropes! → Will prove below.

F.b.d. of the back pack.



$\vec{F} = m\vec{a} = 0$

examine x component and y component of $\vec{F}$ seperately.

Each should be zero!

$(\vec{F})_x = T_1 \cos\theta - T_2 \cos\theta = 0 \Rightarrow T_1 = T_2$! (call it T !)

$(\vec{F})_y = T_1 \sin\theta + T_2 \sin\theta - mg = 0$

Setting $T_1 = T_2 = T$

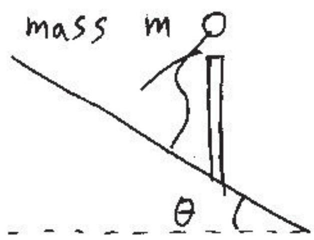
$\Rightarrow \quad 2T \sin\theta = mg$

$$\boxed{T = \frac{mg}{2\sin\theta}} = \ldots = 220\ N$$

$\theta = 22°$

$m = 17\ kg$

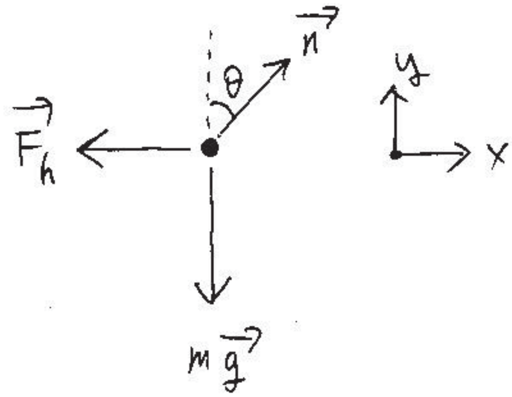
Ex 5.3 of text   Restraining a skier



$\leftarrow \vec{F}_h$ by the gate on the skier

$F_h = ?$

F.b.d. of the skier



$\vec{a} = 0 \rightarrow \vec{F} = 0$

$(\vec{F})_x = n\sin\theta - F_h = 0$

$\therefore F_h = n\sin\theta \quad \cdots (\text{i})$

$(\vec{F})_y = n\cos\theta - mg = 0$

$\therefore n = \dfrac{mg}{\cos\theta} \quad \cdots (\text{ii})$

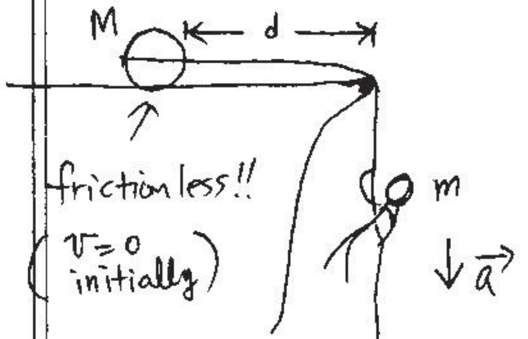
Plug (ii) into (i) $\Rightarrow$ $\boxed{F_h = mg\tan\theta}$

$= \cdots = 340\,\text{N}$

$m = 60\,\text{kg}$

$\theta = 30^\circ$

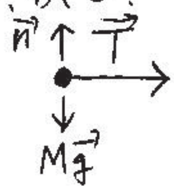
- Ex 5.4 of text **Rock** and the climber



$a = ?$

How much time before the rock starts fall ?

① F.b.d. of the rock

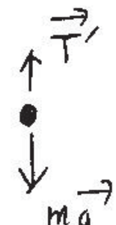


$\vec{n} + M\vec{g} = 0$ (no vertical motion!)

$\vec{T} = M\vec{a}$

magnitude $T = Ma$ --- (i)

② F.b.d. of the climber



$m\vec{g} + \vec{T}' = m\vec{a}$

magnitude $mg - T' = ma$ --- (ii)

③ Neglect the rope's mass. $\Rightarrow$ $T' = T$ tension preserved

Rope acts as a "tension propagator."

Important!!

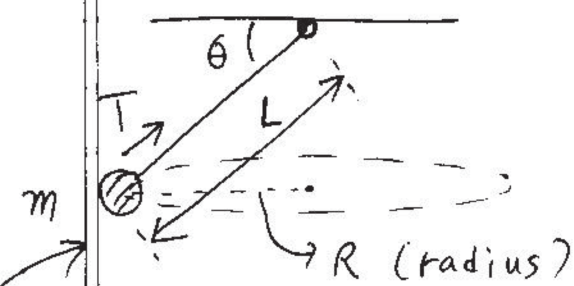
We may encounter some exceptional case later, but for most cases massless rope means that the tension on one end is equal in magnitude to the tension on the other end. Direction can/does change! An exception is when the rope wraps around a massive pulley.

Plug (i) into (ii) ($T' = T$) $\Rightarrow mg - Ma = ma$

$\Rightarrow \boxed{a = \frac{m}{m+M} \cdot g}$

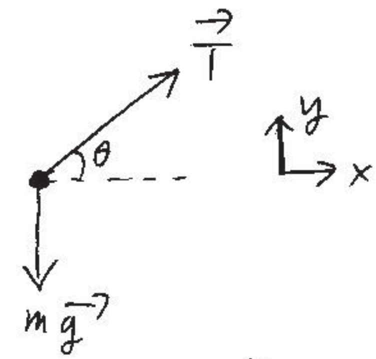
Time $t_F = ?$ $\frac{1}{2} a t_F^2 = d$ $\Rightarrow \boxed{t_F = \sqrt{\frac{2d}{a}}}$

Ex 5.5 of text — Whirling a ball on a string



ball's speed ?
Tension T ?

F.b.d of the ball
(for <u>this</u> instant)



No acceleration along the y direction

$$T \sin\theta = mg \quad \text{---- (i)}$$

Centripetal acceleration along the x direction

$$T \cos\theta = m a_c = m \frac{v^2}{R} \quad \text{--- (ii)}$$

From (i), $\boxed{T = \dfrac{mg}{\sin\theta}}$

Plug this into (ii)

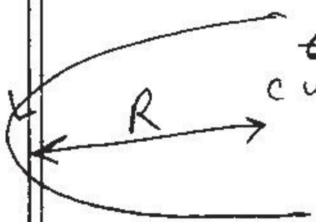
$$m \frac{v^2}{R} = mg \cot\theta \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Note that $R = L \cos\theta$

$$v^2 = \cos^2\theta \, \frac{gL}{\sin\theta}$$

$$\boxed{v = \cos\theta \sqrt{\frac{gL}{\sin\theta}}}$$

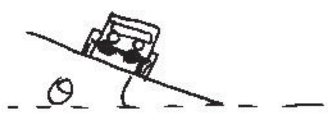
• Ex 5.6 of text — Engineering a road



~~curved~~ curved road (assume no friction)

R: radius

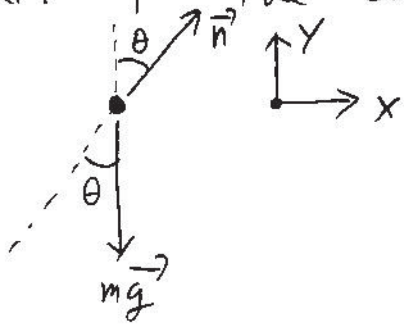
What's the required slope θ of the road?



R, v given $\Rightarrow$ $\theta = ?$

200 m, 25 m/s

F.b.d. of the car



x: direction of the centripetal accel. a_c

y: no acceleration along this direction

$$\vec{F} = m\vec{a}$$

$$(\vec{F})_y = n\cos\theta - mg = 0 \qquad n = \frac{mg}{\cos\theta} \quad \cdots (i)$$

$$(\vec{F})_x = n\sin\theta = m \cdot \underbrace{\frac{v^2}{R}}_{= a_c} \quad \cdots (ii)$$

Plug (i) into (ii),

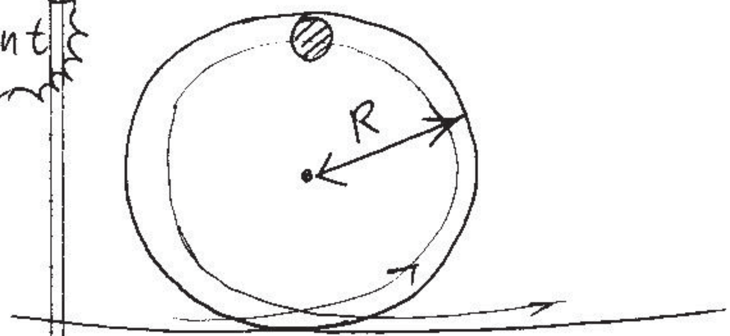
$$mg\tan\theta = m \cdot \frac{v^2}{R}$$

$$\therefore \tan\theta = \frac{v^2}{gR}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)} = \cdots = 18^\circ$$

Important

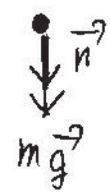
- Ex 5.7 of text, Loop the loop !



$R = 6.3$ m

The minimum speed at top of the loop ?

F.b.d. of the car at top



$$mg + n = m\frac{v^2}{R}$$

Minimum v occurs when $n = 0$

$$\boxed{v = \sqrt{\cancel{m} g R}}$$

Easy to calculate but need to appreciate physics ! (a little like the Tarzan crossing a river problem !)

physics : fast speed → press the track → track pushes back at the car → centripetal acceleration. If the speed is not fast enough, this is not possible.

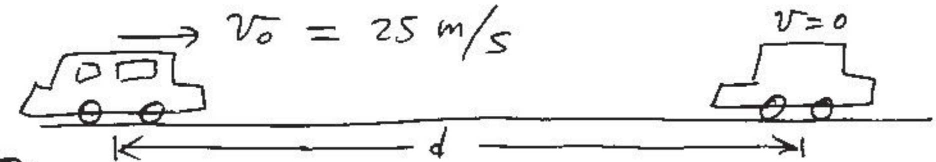
• Ex 5.8 of text Stopping a car

tire and road surface

Static friction coefficient $\mu_s = 0.89$

Kinetic " " $\mu_K = 0.61$

$\vec{v_0} = 25\ m/s$ $\quad v = 0$



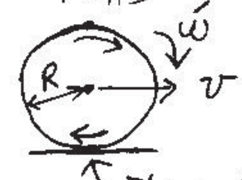
minimum of d ?

①

② d when car is skidding.

important

When a wheel rolls, the contact point is static! $\quad v = \omega R$

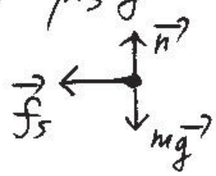


The wheel is not moving relative to the road at this point.

(Static friction applies!)

① minimum d is when the ball rolls normally and thus the maximum friction $\mu_s n = \mu_s mg$ applies. $\Rightarrow a = \mu_s g \Rightarrow \boxed{d_{min} = \dfrac{v_0^2}{2\mu_s g}}$

(F.b.d. of car



$\begin{cases} f_s = \mu_s n \\ n = mg \ (\text{no vertical motion}) \end{cases}$

$\therefore f_s = \mu_s mg$

$a = \mu_s g \Leftarrow ma = \mu_s mg \Leftarrow \vec{F} = m\vec{a}$)

② $a = \mu_K g \Rightarrow \boxed{d = \dfrac{v_0^2}{2\mu_K g}}$

$d > d_{min}$ since $\mu_K < \mu_s$

Ex 5.9 of text Curved road (again)

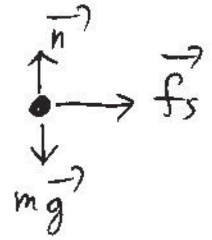
Road with radius $R = 73$ m

$\mu_s = 0.88$ (dry road)

0.21 (snow covered)

$v_{max} = ?$

F.b.d. of the car



No vert. motion

$n = mg$ --- (i)

$f_s \leq \mu_s n$

Horizontally, f_s gives you the centripetal acceleration necessary for you!! It's the only force to save you!!

$$f_s = m\frac{v^2}{R} \leq \mu_s n \underset{\uparrow (i)}{=} \mu_s mg$$

$$\therefore \quad v \leq \sqrt{\mu_s g R}$$

$$\therefore \quad \boxed{v_{max} = \sqrt{\mu_s g R}} = \cdots =$$

dry road
25 m/s

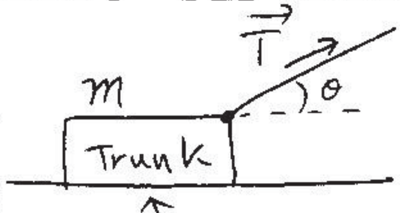
12 m/s
snow covered

Ex 5.10 of text

$$\mu_s = \tan\theta_c$$ (Same as example 5 of LN 08)

θ_c = the angle at which the object starts to slide.

- Ex 5.11 of text

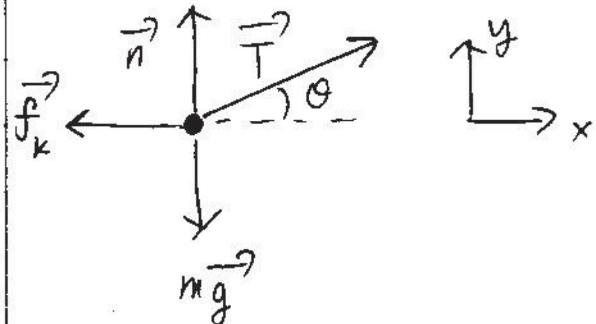


μ_k (kinetic friction coefficient)

Trunk is dragged at a constant ~~speed~~ velocity.

$T = ?$

F.b.d. of the trunk



$\vec{F} = 0$ for const. velocity

- y component of net force

$T \sin\theta + n = mg \quad \cdots (i)$

- x component of net force

$T \cos\theta = \mu_K n \quad \cdots (ii)$

$(i) \times \mu_K + (ii)$

$\Rightarrow \quad T(\mu_K \sin\theta + \cos\theta) = mg\,\mu_K$

$$\Rightarrow \quad \boxed{T = \frac{\mu_K}{\mu_K \sin\theta + \cos\theta} \cdot mg}$$