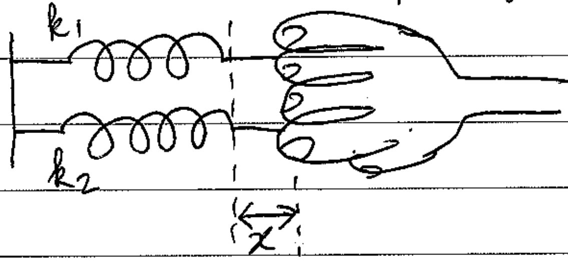
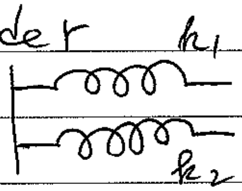
# Springs in parallel (Compound springs)

effective spring constant? $k = k_1 + k_2$

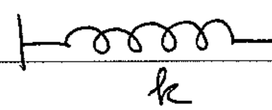



What does "effective spring constant" mean?

Consider




as one object




Why $k = k_1 + k_2$ ?

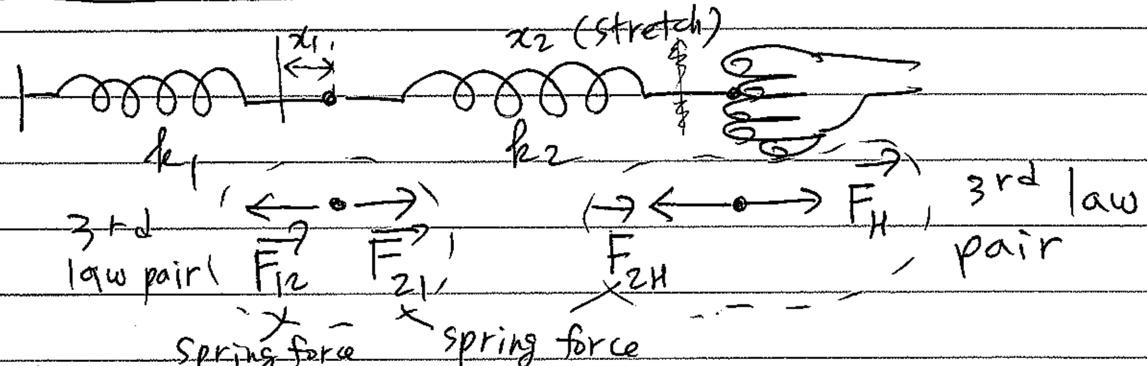
The total spring force exerted on hand (magnitude)

$= k_1 x + k_2 x$

$= (k_1 + k_2)\, x$

Thus $\boxed{k = k_1 + k_2}$

## Springs in series (compound springs)




Magnitude of force $\quad k_1 x_1 = k_2 x_2 \quad k_2 x_2 = F_H$

equal by the 3rd law ①

equal by Hooke's law

equal by the 3rd law ②

The spring force on hand? $\quad$ magnitude $= F_H$

The total amount of elongation? $\quad x_1 + x_2$

So, the effective spring constant

$$k = \frac{\text{spring force}}{\text{deformation}} = \frac{F_H}{x_1 + x_2} \underset{\text{use ②}}{=} \frac{k_2 x_2}{x_1 + x_2} \quad \cdots ③$$

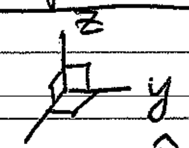
Using ① $\Rightarrow x_1 = \frac{k_2}{k_1} x_2$

Plug this into ③ $\Rightarrow$

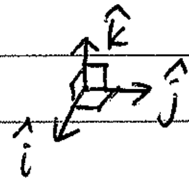
$$k = \frac{k_2 x_2}{\frac{k_2}{k_1} x_2 + x_2} = \cdots = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

$$\boxed{k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}}$$

# Vector product

Consider  coordinate system



Unit vectors  .



<u>Define</u> ① $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

("cyclic permutations")

② $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (non-commuting!)

Using these two definitions and the usual associative rules, one can show

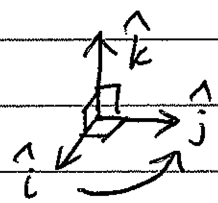
for $\vec{A} = (A_x, A_y, A_z)$

Components

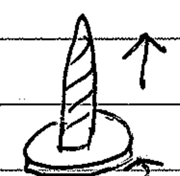
$\vec{B} = (B_x, B_y, B_z)$

How you can calculate $\vec{A} \times \vec{B}$.

$$\Rightarrow \vec{A} \times \vec{B} = (A_y B_z - B_y A_z,\ A_z B_x - B_z A_x,\ A_x B_y - B_x A_y)$$

① means "right-hand rule" 



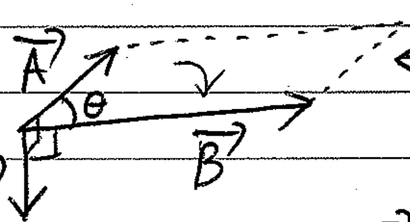


right handed screw

② means "anti-commutative"

geometric

One can show that $|\vec{A} \times \vec{B}| = AB\sin\theta$





← area of the parallelogram

The direction of $\vec{A} \times \vec{B}$ : "right-hand rule"

Note that $\vec{A} \times \vec{B} \perp \vec{A}, \vec{B}$