

## Lecture 7

### Dynamics – Newton’s 2<sup>nd</sup> Law

Weight, tension force, free body diagram, inertial reference frame

#### Kinematics ... and then dynamics

The big lesson that we learn here is that *if there is acceleration (or momentum change, strictly speaking) then there must be a net force*. Note that in this sentence acceleration comes first. The acceleration is a kinematical concept. As we consider forces, we now go into the realm of dynamics. Scientists do not presume that they know force *a priori*. Rather, a force is discovered by studying motions of a certain kind and identifying a force that explains all of them. After some forces are discovered, a more general force that unifies a set of those forces may be discovered later on. Thus force is a concept that is in constant development. On the other hand, the motion (e.g. the acceleration) is in the domain of our senses, and we firmly know what it means. *So, put motion first and then think force next, not the other way around!*

#### Newton’s 2<sup>nd</sup> law and 1<sup>st</sup> law

While Newton’s first law has historical significance, it is just a part of the 2<sup>nd</sup> law, which is

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{where } \vec{p} = m\vec{v} \text{ (momentum)} \quad (7.1)$$

Here we are defining two vector quantities, momentum ( $\vec{p} = m\vec{v}$ ) and  $\vec{F}$  a net force, or the total force.  $m$  is the mass. This law means that “if momentum changes, then there must be a net force.”

For the most part, we will consider the cases where the mass is constant:

$$\vec{F} = m\vec{a} \quad (\text{if mass is constant}) \quad (7.2)$$

The meaning of this equation is that if there is acceleration then there must be a net force. (This sentence is well worth repeating!)

The unit of mass is kg, and the unit of force is kg m/s<sup>2</sup>, from (7.1) and (7.2). There is a special name for this SI unit of force: **Newton (N)**, i.e. **1 N = 1 kg m/s<sup>2</sup>**.

*Newton’s first law* is the special case of (7.2) when  $\vec{F} = 0$ . When the net force is zero, then it follows that the acceleration is zero, which means that  $\vec{v} = \text{constant}$ . In this case, if the object is moving, then its velocity remains unchanged, while if the object is at rest, then it remains at rest. This is Newton’s first law, called *the law of inertia*. **Appreciate the thought experiment of Galileo (Figure 4.1 of the textbook), which came before, and motivated, Newton’s first law.**

This mass  $m$  in the above equations is called the *inertial mass* because the acceleration for a given force is inversely proportional to it ( $\vec{a} = \vec{F}/m$ ). Imagine trying to push a Sumo wrestler or a 5 year old child.

The difference of experience arises because of the large difference in the inertial masses of the wrestler and the child.

*Example 4.1 of the textbook is basic, and should be mastered.*

## Force and net force

There are three fundamental forces known: gravitational force, electro-weak force and strong force. Weak force and strong force explain the radioactivity and how protons or neutrons are formed. In this course, only the gravitational force and the electromagnetic force (part of the electro-weak force) are relevant. The gravitational force is the attraction between two masses. The electromagnetic force is the force between charged particles (electrons, protons) that make up atoms.

It is easy to understand that when things come into contact they exchange influences – they exchange forces. Most forces are like this: frictional force, tension force, spring force, normal force, etc. All of these are, microscopically, electro-magnetic forces, arising from interactions between electrons and/or protons. However, note that electromagnetic forces can also come into play when things are not touching each other. For instance, if you rub a plastic plate and put it above your head, your hair stands up, even though the plastic and the hair are not touching. Another example is the magnetic force between two current carrying wires. You will learn about these in 6B and 6C. Famously, the gravitational force is the only force that acts between large objects on the scale of the Universe. The gravitational force acts between two objects, whether they are in contact or not.

Having laid down this background information, there is one maxim that will be *very useful* for this course (6A). This has to do with the fact that we will not deal with any non-contact force other than the gravitational force. This means, as far as 6A is concerned,

$$\text{No contact, no force, except gravity. (in 6A materials)} \quad (7.3)$$

It turns out that the gravitational force is very small between everyday objects, say books or people or cars, in comparison to the gravitational force between the Earth and an everyday object. So, it is usually safe just to ignore the gravitational force between everyday objects. Which means that *if two every day objects are not touching each other there is no force between them!*

What is a **net force**? It is simply the sum of all forces acting on an object (which is often called a *body* in Newtonian mechanics). It's the same as **the total force**. Keep in mind that the force is a *vector*, and so the sum here means a *vector sum*!

**Example 1. [Weight, mass]** How do we know that there is gravity?

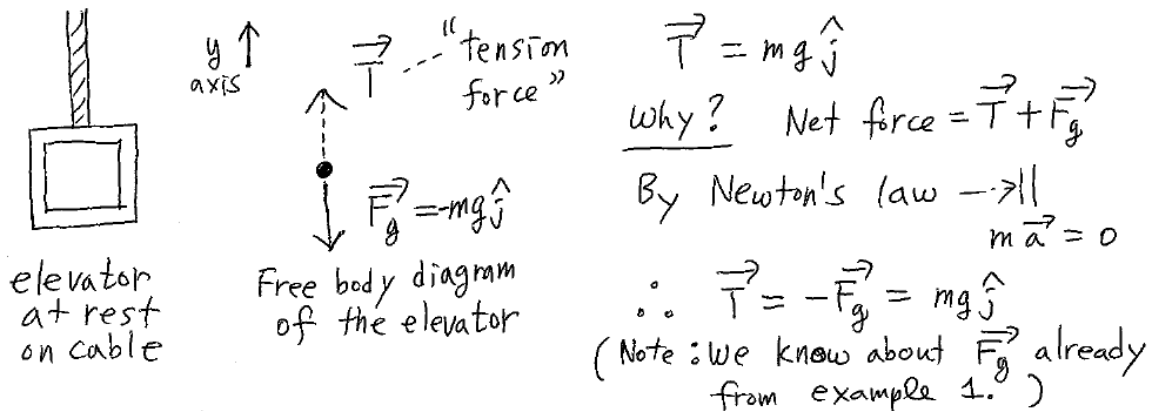
*Solution:* We know gravity since, e.g., when we jump from a high place we know it can hurt. The reason that it can hurt is because we accelerate on the way down. So there must be some force while we are in the air! That is the gravitational force. As we already learned (free fall), near the surface of the Earth, the gravitational force is given by  $-mg\hat{j}$ , if the y axis is taken to point up. In physics, the **gravitational force on an object** is defined as the **weight** of the object. So, the weight is a force. Note that in everyday life, mass and weight are used interchangeably! In physics, they are *not*! They are distinct, as summarized in the following table.

	Nature	Vector? Scalar?	Unit
mass	<i>Intrinsic property</i> (an object's mass is a single value no matter what)	Scalar	kg
weight	<i>Extrinsic property</i> (an object's weight can change in different environments, like different planets or different altitudes)	Vector	N(ewton)

Example 4.2 of textbook is basic, and should be mastered.

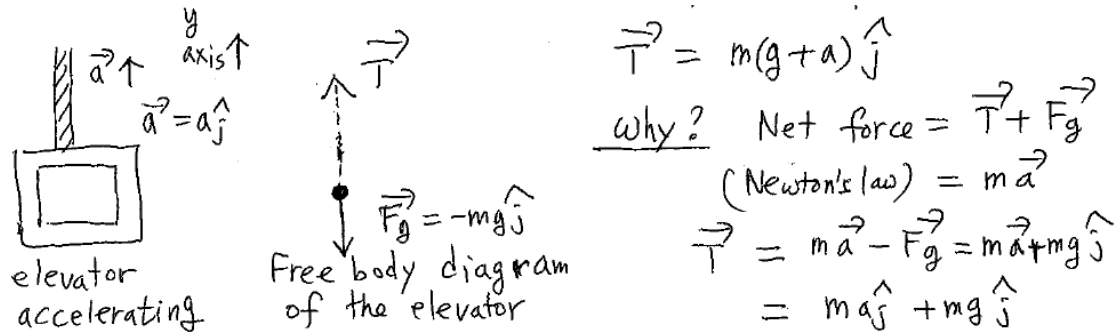
**Example 2. [Tension force, free body diagram]** (cf. Example 4.3 of text). A 740 kg elevator is attached to a cable. (1) When the elevator is at rest, what is the force exerted by the cable on the elevator? (2) When the elevator is moving up at a constant velocity 10 m/s? (3) When moving down at a constant velocity 10 m/s? (4) When accelerating up at  $1.1 \text{ m/s}^2$ ? (5) When accelerating down at  $1.1 \text{ m/s}^2$ ? (6) When accelerating down at  $g = 9.8 \text{ m/s}^2$ ?

*Solution:* First of all, how do we know that the cable exerts a force on the elevator? This comes from Newton's 2<sup>nd</sup> law. In order to apply Newton's 2<sup>nd</sup> law, a **free body diagram** is very useful. The following figure illustrates this point. As shown, the free body diagram of an object is a diagram in which that object is represented as a point (black dot below) and all forces acting on it are drawn as arrows (vectors). **Three things to keep in mind when drawing a free diagram:** (1) The free body diagram of *what*? (2) *No contact, no force, except gravity* (for 6A)! (3) Don't include  $m\vec{a}$ , which is *not* a force.



Note that in this free body diagram of the elevator, we know that there must be a weight force (from the above example). Since we know that the elevator is at rest, we must conclude, based on Newton's law, that there must be another force that cancels the weight force! That other force is the force that is exerted by the cable, since the cable is the only thing that is in contact. The force that is exerted by a cable or a string on an object that is attached to it is called a **tension force**.

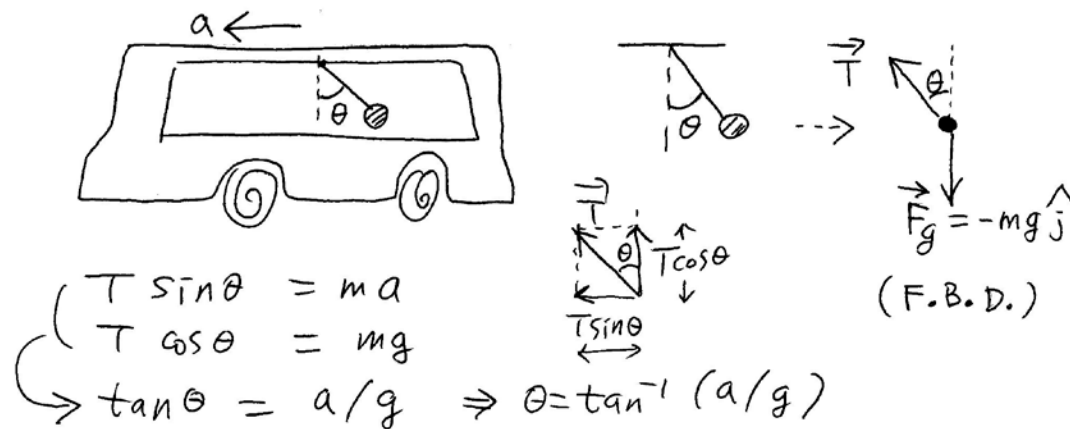
We just proved that there must be such a force! At the same time we got the common answer for (1,2,3):  $\vec{T} = mg\hat{j} = 7.3\hat{j} \text{ kN}$ . Why (2) and (3) have this same answer? Because  $\vec{a} = 0$ . Now, for (4, 5,6) the following analysis applies.



For (4),  $a = 1.1 \text{ m/s}^2$  and so  $T = m(g+a) = 8.1 \text{ kN}$  (up). For (5),  $a = -1.1 \text{ m/s}^2$  and so  $T = 6.4 \text{ kN}$  (up). For (6),  $a = -g$ , and so  $T = 0 \text{ N}$ !

**Example 3. [Inertial reference frame]** In a bus, a mass is attached by a string to the ceiling of the bus. The mass is at rest. Then, the bus starts to accelerate at  $1.2 \text{ m/s}^2$ . During this acceleration the mass is observed to “be pushed back” and make a certain angle relative to the vertical angle, but remains at rest in the bus reference frame. What is the angle? Is the Newton’s law valid in the bus?

*Solution.* From the diagram below,  $\theta = \tan^{-1}(a/g) = 0.12 \text{ radians} = 7.0 \text{ degrees}$ . As for the second question, the answer is no. Why? From the free body diagram below the net force is not zero. On the other hand, the mass is not moving, so  $m\vec{a} = 0$  in the bus reference frame! So,  $\vec{F} = m\vec{a}$  is violated.



A reference frame in which Newton’s law is valid is defined as an **inertial reference frame**. Any accelerating reference frame is not an inertial reference frame. This includes the bus reference frame in the above example, or any rotating reference frame (remember “circular motion  $\rightarrow$  centripetal acceleration”). You might think that this is very strange – the Earth, since it is rotating, is not an inertial reference frame and we are doing all these physics experiments to study Newton’s laws? Indeed, strictly speaking the Earth is not an inertial reference frame. However, for your lab works, all “non-inertial effects” are quite small.