STATICS

Chap. 12 of Wolfson

What you will learn

- Condition for static equilibrium
- How to *efficiently* solve a static eq. problem
- Different types of equilibrium

Static equilibrium

$$\sum \vec{F}_{i,ext} = 0 \quad \text{and} \quad \sum \vec{\tau}_{i,ext} = 0$$

- The net torque should be zero, around **any** axis.
- However, if the net torque is zero for one axis of rotation, then it is zero for *any* axis of rotation, as long as the net force is zero (as in here). So, we are free to make any convenient/efficient choice of the axis of rotation!

Proof: $\sum \vec{\tau}_{i,A,ext} = \sum \vec{r}_{i,A} \times \vec{F}_{i,ext} = \sum (\vec{r}_{i,B} + \vec{r}_{AB}) \times \vec{F}_{i,ext}$
$= \sum \vec{\tau}_{i,B,ext} + \vec{r}_{AB} \times \sum \vec{F}_{i,ext}$ (now use $\sum \vec{F}_{i,ext} = 0$)
$= \sum \vec{\tau}_{i,B,ext}$
where A and B are two reference frames.

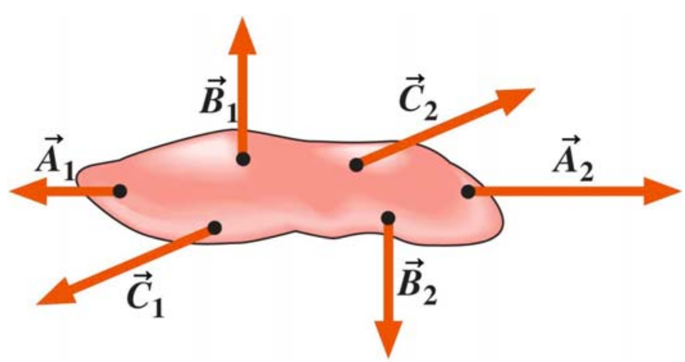
Quiz

- The figure shows three pairs of forces acting on an object. Which pair, acting as *the only forces* on the object, results in static equilibrium?

A. Force pair $\vec{A}_1 + \vec{A}_2$

B. Force pair $\vec{B}_1 + \vec{B}_2$

C. Force pair $\vec{C}_1 + \vec{C}_2$



The center of gravity

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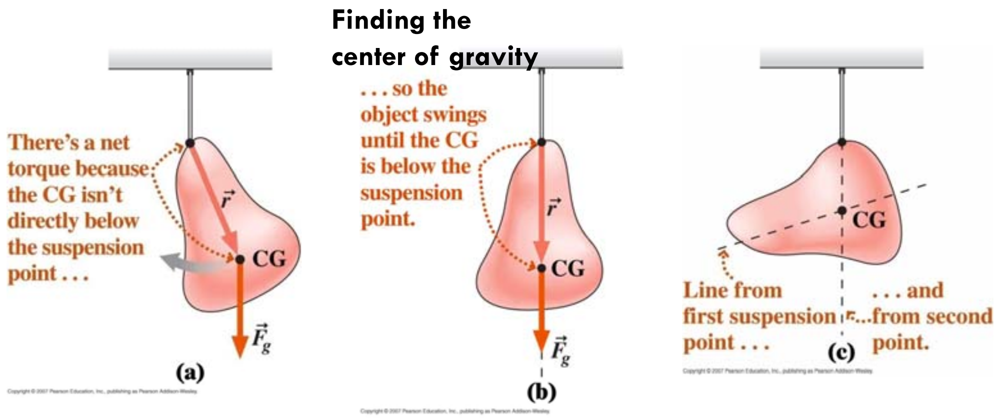
- Coincides with the center of mass, when the gravitational field is uniform.
- This is the point at which the gravity is applied!



Quiz

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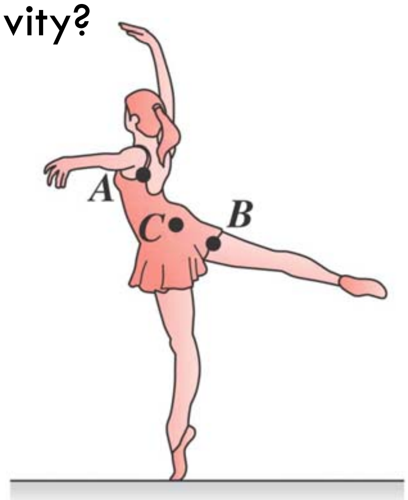
- The dancer in the figure is balanced; that is, she is in static equilibrium. Which of the three lettered points could be her center of gravity?

A. Point *A*

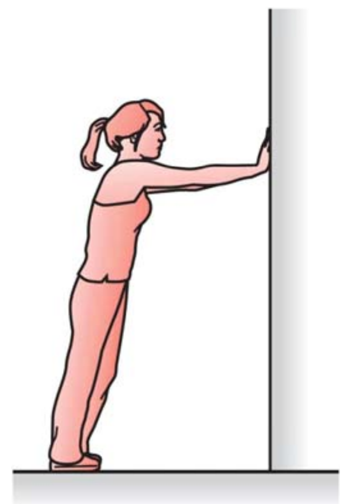
B. Point *B*

C. Point C

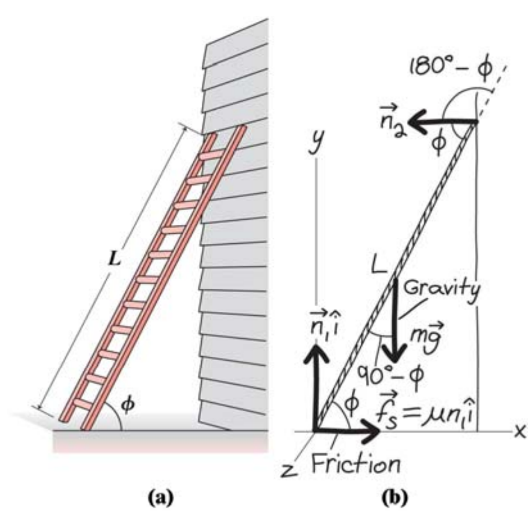


Quiz

- The figure shows a person in static equilibrium leaning against a wall. Which one of the following must be true?
  A. There must be a frictional force at the wall, but not necessarily at the floor.
  B. There must be a frictional force at the floor, but not necessarily at the wall.
  C. There must be frictional forces at both wall and floor.



At what angle will the ladder slip?

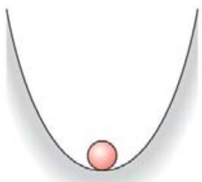


$\sum F_x = 0: \ \mu n_1 = n_2$
$\sum F_y = 0: \ n_1 = mg$

Smart choice of the origin for torque calculation: Where the ladder touches the floor.
$\sum \tau = 0: \ L n_2 \sin\phi = \frac{L}{2} mg \cos\phi$

$\tan\phi = 1/(2\mu)$
$\phi = \arctan(\frac{1}{2\mu})$ is the answer.

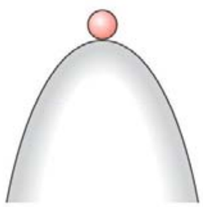
Not all equilibrium states are stable!


(a)

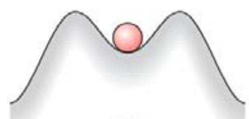
Stable equilibrium: disturbed ball will return to equilibrium


(c)

Neutrally stable equilibrium


(b)

Unstable equilibrium: disturbed ball will leave original equilibrium


(d)

Metastable or conditionally stable or "locally stable" equilibrium: ball returns for small disturbances, but not for large ones

Potential energy and equilibrium

Consider $U = U(q)$
$q = x$ (mass on spring, e.g.) or
$q = \theta$ (pendulum, bear on high wire, e.g.)

Equilibrium: $dU/dq = 0$

Stable Equilibrium: $dU/dq = 0, d^2U/dq^2 > 0$ (meta or not)
Unstable Equilibrium: $dU/dq = 0, d^2U/dq^2 < 0$
Neutrally Stable Equilibrium: $dU/dq = 0, d^2U/dq^2 = 0$

Quiz

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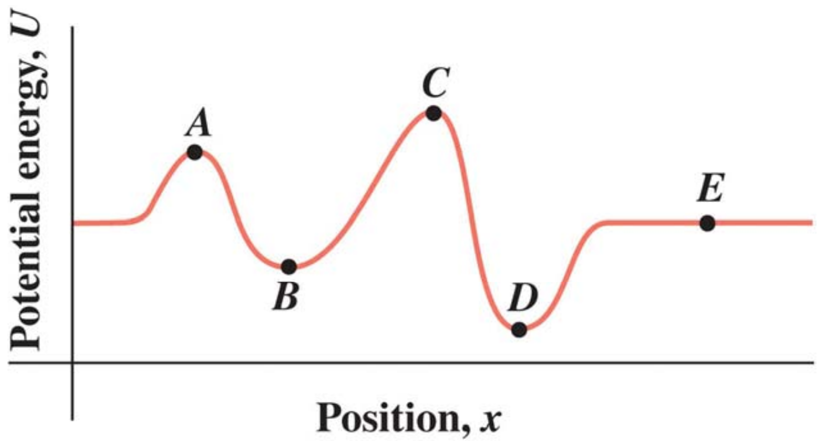
- Which of the labeled points in the figure is in metastable equilibrium?



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Concluding with ? … !

- What are the conditions for static equilibrium?
- For an object to stand, the projection of the center of gravity should fall within the basal contact area. Why?
- How do you analyze the stability of a leaning ladder?
- How do you tell the existence and the stability of static equilibrium from the potential curve such as U(x) or U(θ)?