MANY PARTICLES AND COLLISIONS

Chap. 9 of Wolfson (cont.)

What was and what will be

- ✓Center of mass
- ✓Motion of center of mass $\boxed{\vec{F}_{net,ext} = \dot{\vec{P}}} = M\ddot{\vec{r}}_{cm}$
- Momentum Conservation
- Elastic and Inelastic Collisions

CM accelerates only by external force

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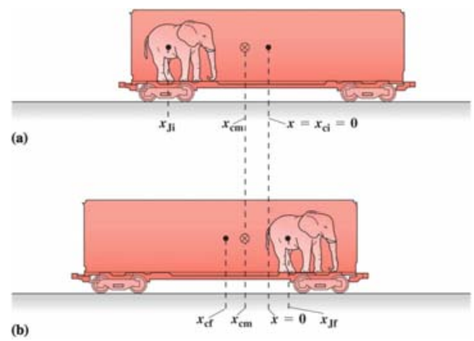
On a frictionless surface, there is a railcar (C) at rest.
Jumbo (J), a 4.8 t elephant, moves by 19 m, in a 15 t railcar.
How much does the railcar move?

Ex. 9.4

Key observation:
There is NO external force!
So, the CM of J+C does not move.



$$M x_{cm} = m_J x_J + m_C x_C$$
$$M \Delta x_{cm} = m_J \Delta x_J + m_C \Delta x_C = 0$$
$$\Delta x_J - \Delta x_C = 19 \text{ m}$$
$$m_J(19\text{m} + \Delta x_C) + m_C \Delta x_C = 0$$
$$\Delta x_C = -\frac{m_J}{m_J + m_C} 19\text{m} = -4.6\text{m}$$

Principle of Momentum Conservation

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- Without external force, the total momentum is conserved!!

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

- The Jumbo problem was one example. Suppose now Jumbo jumps out, with a certain velocity, then the car will gain a velocity in the other direction. This is the principle of rocket propulsion/thrust!!

Please review Ex.'s 9.5, 9.6, 9.7!

Quiz

A 500-g fireworks rocket is moving with velocity $\vec{v} = 60\hat{\boldsymbol{j}}$ m/s at the instant it explodes. If you were to add the momentum vectors of all its fragments just after the explosion, what would be the result?

A. $\vec{v} = 60\hat{\boldsymbol{j}}$ kg·m/s

B. $\vec{v} = 30\hat{\boldsymbol{j}}$ kg·m/s

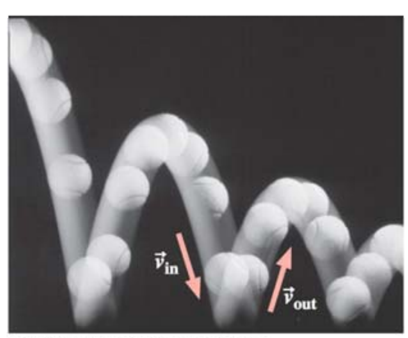
C. $\vec{v} = 60000\hat{\boldsymbol{j}}$ kg·m/s

D. $\vec{v} = 30000\hat{\boldsymbol{j}}$ kg·m/s

Collision

http://www.specialised-imaging.com/

http://farm4.static.flickr.com



Collision

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- Very short and violent interaction between objects
- Total momentum $\vec{P}$ is **always conserved** (assuming very short collision)
- Total kinetic energy K is not necessary conserved

  Elastic collision (K is conserved)

  Inelastic collision (K is not conserved)

  Totally Inelastic (two becomes one; or one becomes two)

Impulse

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This slide is for optional reading.

- During a collision, objects exchange momentum.

Δt = duration of collision

$\Delta \vec{p} = \vec{F}_{ave}\Delta t$ (from Newtons 2nd law)

$\vec{F}_{ave}$ = average force the object experiences during collision

Warning: $\vec{p}$ is for *an* object, not total!

Impulse: $\boxed{\vec{J} \equiv \vec{F}_{ave}\Delta t}$

$\Delta \vec{p} = \vec{J}$

- "Pulling the table-cloth" trick works by minimizing the impulse (i.e. by minimizing the time).

Quiz

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- Two skaters toss a basketball back and forth on frictionless ice. Which one of the following does not change?

A. The momentum of an individual skater
B. The momentum of the system consisting of one skater and the basketball
C. The momentum of the basketball
D. The momentum of the system consisting of both skaters and the basketball

Ballistic Pendulum
A totally inelastic collision

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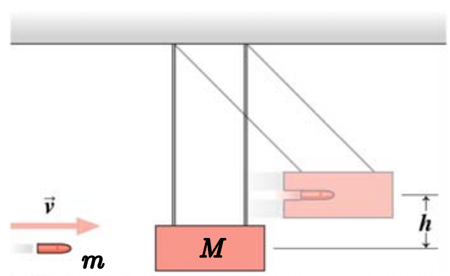
- If block+bullet goes up by *h*, what was *v*?
  1. Two becomes one on bullet being buried: totally inelastic collision – only the total momentum is conserved during this collision.
  2. Then, b+b swings – a "simple" pendulum problem: total mechanical energy is conserved.

Ex. 9.10



1. $mv = (M+m)v_f$
2. $(M+m)gh = \frac{1}{2}(M+m)v_f^2$

From 1, $v = \frac{M+m}{m}v_f$.
From 2, $v_f = \sqrt{2gh}$, and so
$v = \frac{M+m}{m}\sqrt{2gh}$

Croquet or Billiard
An elastic collision in 2D

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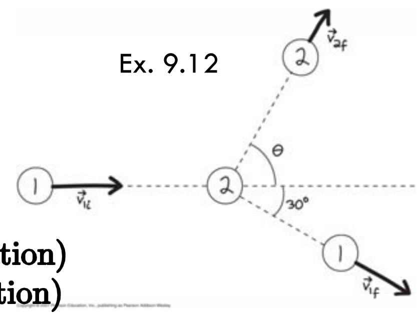
- In an elastic collision, a ball strikes a stationary ball of the same mass, and comes out at 30 degrees. In what direction is the other ball moving?



$m\vec{v}_{1i} = m\vec{v}_{1f} + m\vec{v}_{2f}$ (momentum conservation)
$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$ (energy conservation)
In general it can be nasty to solve these two conservation equations.
In this case, it is simple since m cancels out. We get
$\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$ – (1)
$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ – (2)
By taking dot product of (1) with itself on both sides,
and subtracting it from (2), we get
$\boxed{\vec{v}_{1f} \cdot \vec{v}_{2f} = 0}$ (i.e. the two outgoing velocities are orthogonal)
And so, $\theta = 60°$ in this case.

Elastic collision in 1D

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- Two unknowns (v_{1f}, v_{2f})

  and two equations $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
  $\frac{1}{2}(m_1v_{1i}^2 + m_2v_{2i}^2) = \frac{1}{2}(m_1v_{1f}^2 + m_2v_{2f}^2)$
- CAN solve!!
- Solution:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

(relative velocity is simply reversed in sign!)

detailed solutions, after some algebra …

$$v_{1f} = \frac{1}{m_1+m_2}[(m_1 - m_2)v_{1i} + 2m_2v_{2i}]$$
$$v_{2f} = \frac{1}{m_1+m_2}[(2m_1)v_{1i} + (m_2 - m_1)v_{2i}]$$

Elastic collision in 1D – Special Cases

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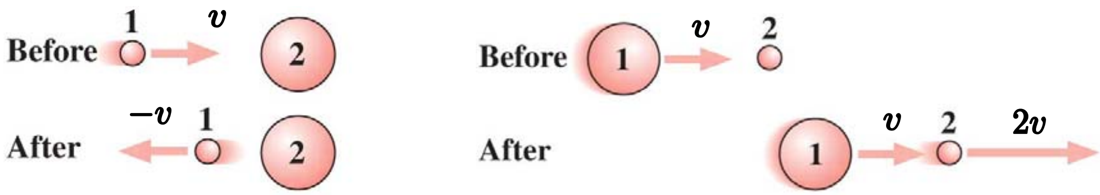
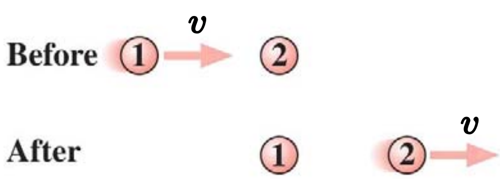


(a) $m_1 << m_2$ (c) $m_1 >> m_2$



(b) $m_1 = m_2$

[Note, optional]
If you understand one of (a) or (c), then the other follows.
How so? For example, to go from (a) to (b), describe the collision in an inertial frame that is moving at the initial velocity of 1, and then swap 1 and 2.

Elastic collision in 2D

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This slide is for optional reading.

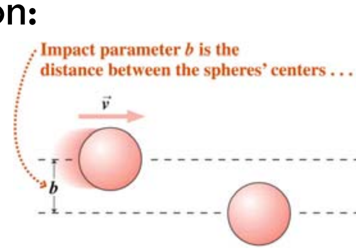
- Four unknowns $(\vec{v}_{1f}, \vec{v}_{2f})$

  and three equations $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

  $\frac{1}{2}(m_1 v_{1i}^2 + m_2 v_{2i}^2) = \frac{1}{2}(m_1 v_{1f}^2 + m_2 v_{2f}^2)$
- CANNOT solve!!
- Need one more piece of information:

  Impact Parameter *b*
  - Non-head-on nature of collision
  - Related to the angular momentum

  or direction of one particle after collision



Concluding with ? … !

- What determines the motion of the center of mass?
- When is the total momentum conserved?
- In what type(s) of collision, is the momentum conserved?
- In what type of collision, is the energy conserved?
- How does a ballistic pendulum work?
- Can you describe three special types of 1D elastic collisions?