

# Many Particles and Collisions

Momentum Conservation

Elastic and Inelastic Collisions

Center of Mass!

Chap. 9 of Wolfson

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## Center of Mass (recap)

### 18.2. DEFINITION. Center of Mass

For an object that consists of discrete points ( $m_i$  and  $\vec{r}_i$ ), the center of mass coordinate  $\vec{R}_{cm}$  is defined as

$$\vec{R}_{cm} \equiv \frac{\sum_i m_i \vec{r}_i}{M}$$

where  $M$  is the total mass  $M = \sum_i m_i$ . Similarly, for a continuous object

$$\vec{R}_{cm} \equiv \frac{\int dm \vec{r}}{M}$$

where  $M = \int dm$ .

**Note : notation change  $\vec{R}_{cm} \rightarrow \vec{r}_{cm}$**

$$\vec{r}_{cm} \equiv \frac{\sum_i m_i \vec{r}_i}{M}$$

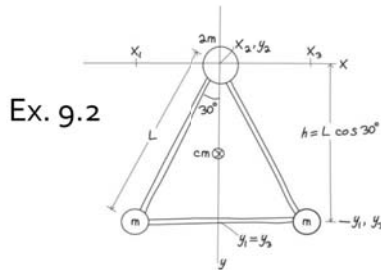
$$\vec{r}_{cm} \equiv \frac{\int dm \vec{r}}{M}$$

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# Calculating Center of Mass

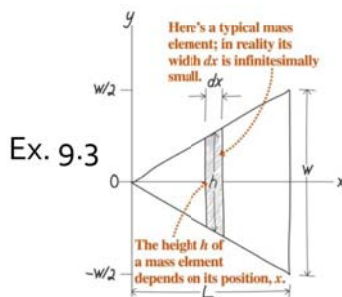
- A “discrete” system of particles



$$x_{cm} = \frac{mx_1 + mx_3}{4m} = \frac{m(x_1 - x_3)}{4m} = 0$$

$$y_{cm} = \frac{my_1 + my_3}{4m} = \frac{2my_1}{4m} = \frac{y_1}{2} = \frac{\sqrt{3}}{4}L$$

- A continuous system of particles



$$dm = \frac{2M}{Lw} h dx = \frac{2M}{Lw} 2 \frac{w}{2L} x dx = \frac{2Mx}{L^2} dx$$

$$x_{cm} = \frac{\int x dm}{M} = \frac{\int_0^L \frac{2Mx^2}{L^2} dx}{M} = \frac{2}{3}L$$

$$y_{cm} = 0 \text{ by symmetry}$$

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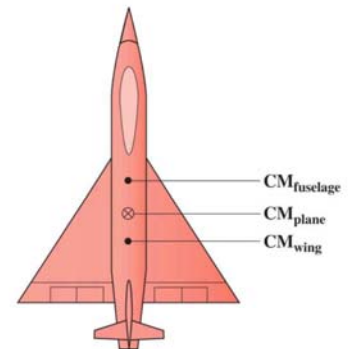
# Center of Mass of a Complex Object

- The center of mass of a composite object can be found from the CMs of its individual parts.

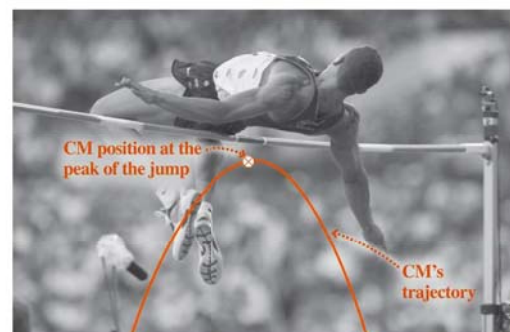
*proof*

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{M} = \frac{\sum_{i_1} m_{i_1} \vec{r}_{i_1} + \sum_{i_2} m_{i_2} \vec{r}_{i_2}}{M} = \frac{M_1 \vec{r}_{cm,1}}{M} + \frac{M_2 \vec{r}_{cm,2}}{M}$$

$$\sum_i m_i = M, \sum_{i_1} m_{i_1} = M_1, \sum_{i_2} m_{i_2} = M_2, M_1 + M_2 = M$$



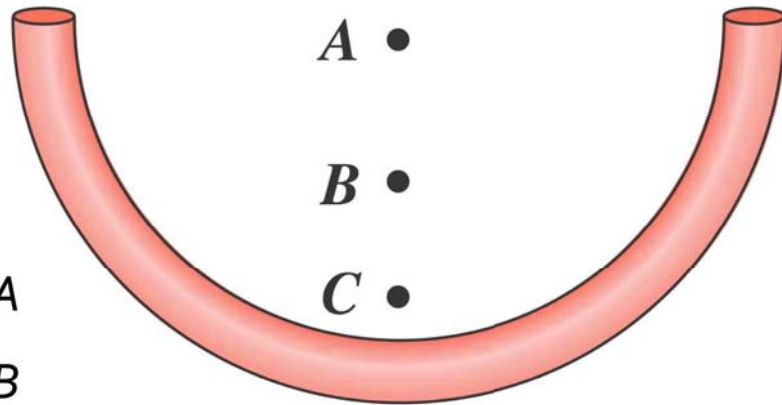
- The high jumper clears the bar, but his CM does not.



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# Quiz

- A thick wire is bent into a semicircle, as shown in the figure. Which of the points shown is the center of mass of the wire?



- A. Point A
- B. Point B
- C. Point C

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# Why is the CM VERY important?

- The CM accelerates only by external force!

Internal forces cancel by Newton's 3<sup>rd</sup> law!

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

- Total Momentum

$$\vec{P} \equiv \sum_i m_i \dot{\vec{r}}_i$$

$$\vec{P} = M\dot{\vec{r}}_{cm}$$

- Total Kinetic Energy

$$K = \frac{1}{2}M|\dot{\vec{r}}_{cm}|^2 + K_{int}$$

$$K_{int} = \frac{1}{2} \sum_i m_i |\dot{\vec{r}}_{i,cm}|^2$$

Internal KE (e.g. rotational)  
Theorem 19.4  
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## CM accelerates only by external force

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$



Independent of any spinning, twisting, flapping, etc. of the skier (or diver, or skater) the trajectory of the **CM** in air is that of the projectile motion of a **point particle** with the total mass **M**!!

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## CM accelerates only by external force

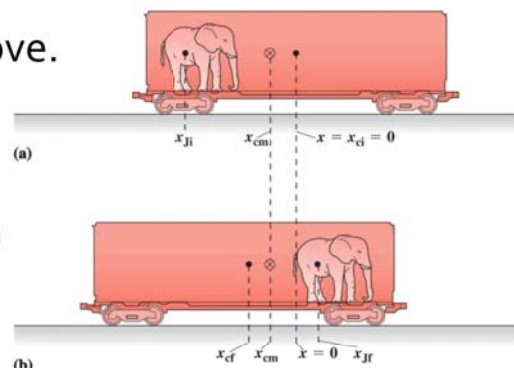
On a frictionless surface, there is a railcar (C) at rest. Jumbo (J), a 4.8 t elephant, moves by 19 m, in a 15 t railcar. How much does the railcar move?

Key observation:

There is NO external force!

So, the CM of J+C does not move.

Ex. 9.4



$$\begin{aligned} Mx_{cm} &= m_J x_J + m_C x_C \\ M\Delta x_{cm} &= m_J \Delta x_J + m_C \Delta x_C = 0 \\ \Delta x_J - \Delta x_C &= 19 \text{ m} \\ m_J(19\text{m} + \Delta x_C) + m_C \Delta x_C &= 0 \\ \Delta x_C &= -\frac{m_J}{m_J + m_C} 19\text{m} = -4.6\text{m} \end{aligned}$$

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# Principle of Momentum Conservation

- Without external force, the total momentum is conserved!!

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

- The Jumbo problem was one example. Suppose now Jumbo jumps out, with a certain velocity, then the car will gain a velocity in the other direction. This is the principle of rocket propulsion/thrust!!



Please review Ex.'s 9.5, 9.6, 9.7!

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## Quiz

A 500-g fireworks rocket is moving with velocity  $\vec{v} = 60\hat{j}$  m/s at the instant it explodes. If you were to add the momentum vectors of all its fragments just after the explosion, what would be the result?

- A.  $\vec{v} = 60\hat{j}$  kg·m/s
- B.  $\vec{v} = 30\hat{j}$  kg·m/s
- C.  $\vec{v} = 60000\hat{j}$  kg·m/s
- D.  $\vec{v} = 30000\hat{j}$  kg·m/s

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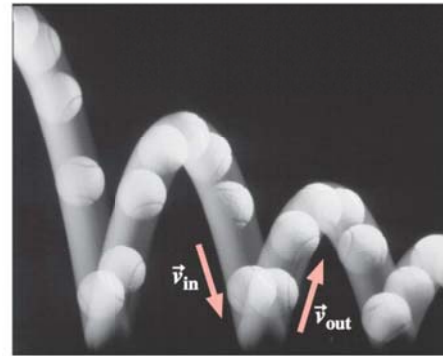
# Collision



<http://www.specialised-imaging.com/>



<http://farm4.static.flickr.com>



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# Collision

- Very short and violent interaction between objects
- Total momentum  $\vec{P}$  is **always conserved** (assuming very short collision)
- Total kinetic energy  $K$  is not necessarily conserved

**Elastic** collision ( $K$  is conserved)

**Inelastic** collision ( $K$  is not conserved)

**Totally Inelastic** (two becomes one; or one becomes two)

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# Impulse

- During a collision, objects exchange momentum.

$\Delta t$  = duration of collision

$\Delta \vec{p} = \vec{F}_{ave} \Delta t$  (from Newton's 2nd law)

$\vec{F}_{ave}$  = average force the object experiences during collision

Warning:  $\vec{p}$  is for *an* object, not total!

**Impulse:**  $\vec{J} \equiv \vec{F}_{ave} \Delta t$

$\Delta \vec{p} = \vec{J}$

- “Pulling the table-cloth” trick works by minimizing the impulse (i.e. by minimizing the time).

# Quiz

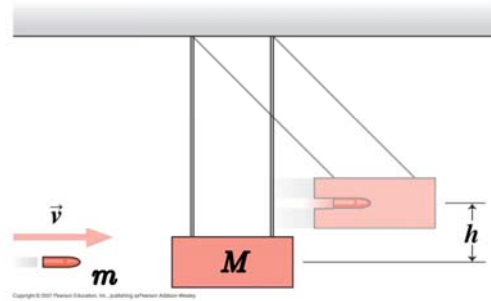
Two skaters toss a basketball back and forth on frictionless ice. Which one of the following does not change?

- A. The momentum of an individual skater
- B. The momentum of the system consisting of one skater and the basketball
- C. The momentum of the basketball
- D. The momentum of the system consisting of both skaters and the basketball

# Ballistic Pendulum (totally inelastic collision)

Ex. 9.10

- If block+bullet goes up by  $h$ , what was  $v$ ?



1. Two becomes one: totally inelastic collision – only the total momentum is conserved.
2. Then, it becomes a “simple” pendulum: total mechanical energy is conserved.

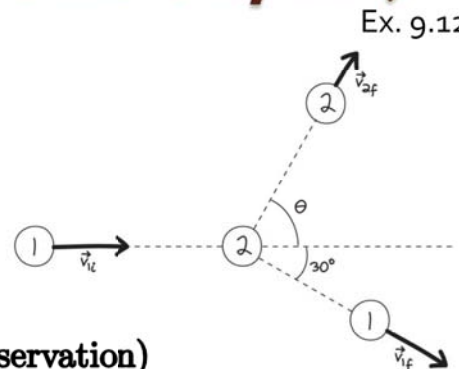
1.  $mv = (M + m)v_f$
2.  $(M + m)gh = \frac{1}{2}(M + m)v_f^2$

From 1,  $v = \frac{M+m}{m}v_f$ .  
 From 2,  $v_f = \sqrt{2gh}$ , and so  
 $v = \frac{M+m}{m}\sqrt{2gh}$

# Croquet (Elastic Collision, 2D)

Ex. 9.12

- In an elastic collision, a ball strikes a stationary ball of the same mass, and comes out at 30 degrees. In what direction is the other ball moving?



$$m\vec{v}_{1i} = m\vec{v}_{1f} + m\vec{v}_{2f} \quad (\text{momentum conservation})$$

$$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 \quad (\text{energy conservation})$$

In general it can be nasty to solve these two conservation equations.

In this case, it is simple since  $m$  cancels out. We get

$$\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f} \quad (1)$$

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 \quad (2)$$

By taking dot product of (1) with itself on both sides, and subtracting it from (2), we get

$$\vec{v}_{1f} \cdot \vec{v}_{2f} = 0 \quad (\text{i.e. the two outgoing velocities are orthogonal})$$

And so,  $\theta = 60^\circ$  in this case.