Many Particles and Collisions

Momentum Conservation

Elastic and Inelastic Collisions

Center of Mass!

Chap. 9 of Wolfson

Center of Mass (recap)

18.2. DEFINITION. **Center of Mass**

For an object that consists of discrete points (m_i and $\vec{r}_i$), the center of mass coordinate $\vec{R}_{cm}$ is defined as

$$\vec{R}_{cm} \equiv \frac{\sum_i m_i \vec{r}_i}{M}$$

where M is the total mass $M = \sum_i m_i$. Similarly, for a continuous object

$$\vec{R}_{cm} \equiv \frac{\int dm \vec{r}}{M}$$

where $M = \int dm$.

Note : notation change $\vec{R}_{cm} \rightarrow \vec{r}_{cm}$

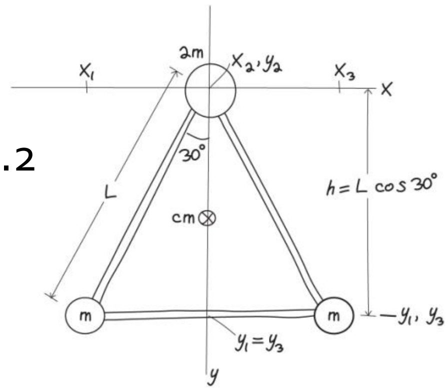
$$\vec{r}_{cm} \equiv \frac{\sum_i m_i \vec{r}_i}{M}$$

$$\vec{r}_{cm} \equiv \frac{\int dm \vec{r}}{M}$$

Calculating Center of Mass

- A "discrete" system of particles

Ex. 9.2

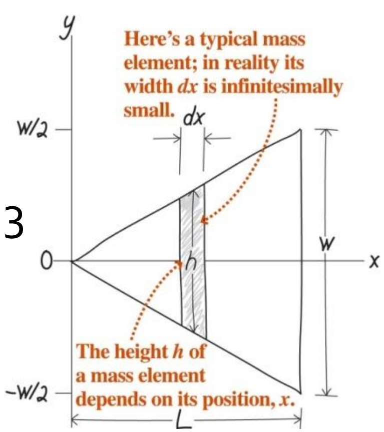


$$x_{cm} = \frac{mx_1 + mx_3}{4m} = \frac{m(x_1 - x_1)}{4m} = 0$$

$$y_{cm} = \frac{my_1 + my_3}{4m} = \frac{2my_1}{4m} = \frac{y_1}{2} = \frac{\sqrt{3}}{4}L$$

- A continuous system of particles

Ex. 9.3



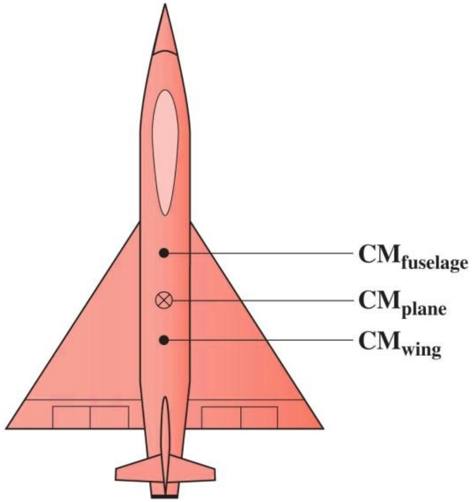
$$dm = \frac{2M}{Lw} h dx = \frac{2M}{Lw} 2 \frac{w}{2L} x dx = \frac{2Mx}{L^2} dx$$

$$x_{cm} = \frac{\int x dm}{M} = \frac{\int_0^L \frac{2Mx^2}{L^2} dx}{M} = \frac{2}{3} L$$

$$y_{cm} = 0 \text{ by symmetry}$$

Center of Mass of a Complex Object

- The center of mass of a composite object can be found from the CMs of its individual parts.

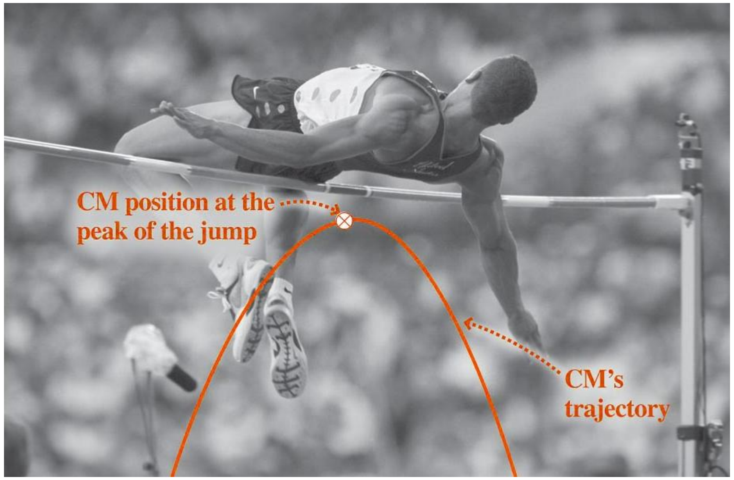


proof

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{M} = \frac{\sum_{i_1} m_{i_1}\vec{r}_{i_1} + \sum_{i_2} m_{i_2}\vec{r}_{i_2}}{M} = \frac{M_1}{M}\vec{r}_{cm,1} + \frac{M_2}{M}\vec{r}_{cm,2}$$

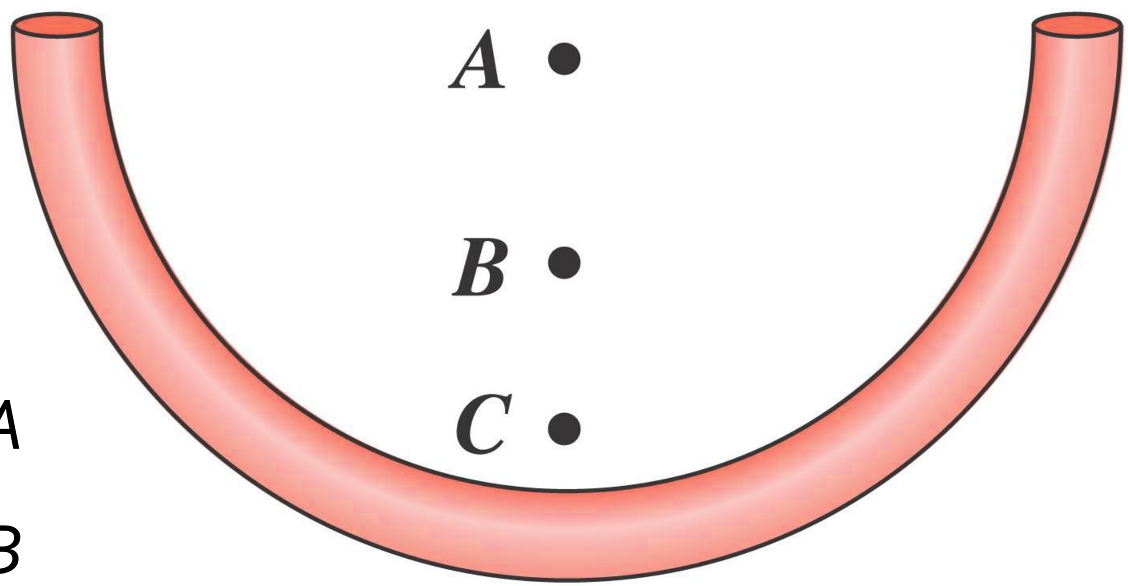
$$\sum_i m_i = M, \sum_{i_1} m_{i_1} = M_1, \sum_{i_2} m_{i_2} = M_2, M_1 + M_2 = M$$

- The high jumper clears the bar, but his CM does not.



Quiz

- A thick wire is bent into a semicircle, as shown in the figure. Which of the points shown is the center of mass of the wire?



A. Point *A*

B. Point *B*

C. Point *C*

Why is the CM VERY important?

- The CM accelerates only by external force!

Internal forces cancel by Newton's 3rd law!

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

- Total Momentum

$$\vec{P} \equiv \sum_i m_i \dot{\vec{r}}_i \qquad \vec{P} = M\dot{\vec{r}}_{cm}$$

- Total Kinetic Energy

$$K = \frac{1}{2}M|\dot{\vec{r}}_{cm}|^2 + K_{int}$$

$$K_{int} = \frac{1}{2}\sum_i m_i |\dot{\vec{r}}_{i,cm}|^2$$

Internal KE (e.g. rotational)
Theorem 19.4
Lec_11-07

CM accelerates only by external force

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

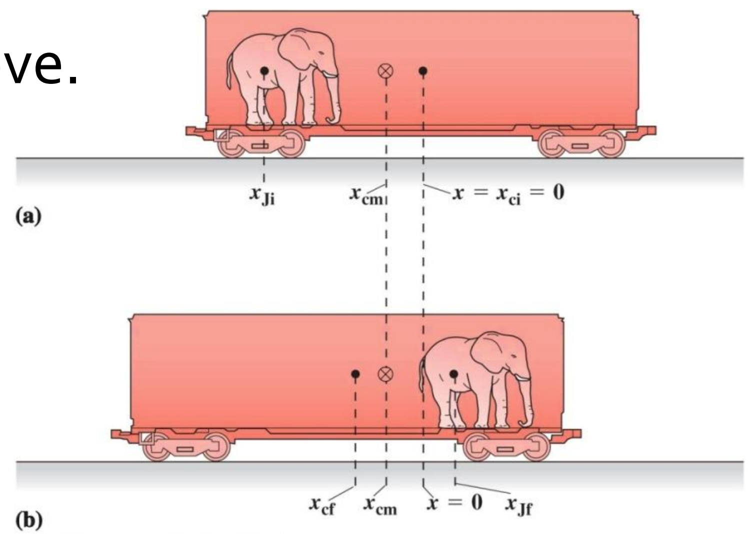
Independent of any spinning, twisting, flapping, etc. of the skier (or diver, or skater) the trajectory of the **CM** in air is that of the projectile motion of a **point particle** with the total mass **M**!!

CM accelerates only by external force

On a frictionless surface, there is a railcar (C) at rest.
Jumbo (J), a 4.8 t elephant, moves by 19 m, in a 15 t railcar.
How much does the railcar move?

Key observation:
There is NO external force!
So, the CM of J+C does not move.

Ex. 9.4



$$M x_{cm} = m_J x_J + m_C x_C$$
$$M \Delta x_{cm} = m_J \Delta x_J + m_C \Delta x_C = 0$$
$$\Delta x_J - \Delta x_C = 19 \text{ m}$$
$$m_J(19\text{m} + \Delta x_C) + m_C \Delta x_C = 0$$
$$\Delta x_C = -\frac{m_J}{m_J + m_C} 19\text{m} = -4.6\text{m}$$

Principle of Momentum Conservation

- Without external force, the total momentum is conserved!!

$$\vec{F}_{net,ext} = M\ddot{\vec{r}}_{cm} = \dot{\vec{P}}$$

- The Jumbo problem was one example. Suppose now Jumbo jumps out, with a certain velocity, then the car will gain a velocity in the other direction. This is the principle of rocket propulsion/thrust!!

Please review Ex.'s 9.5, 9.6, 9.7!

Quiz

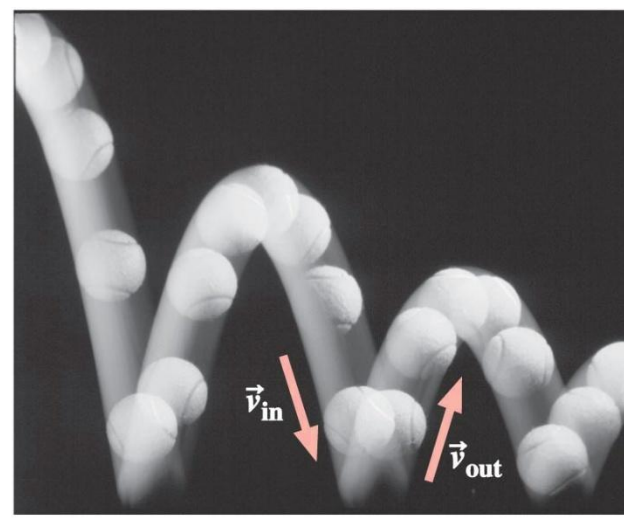
A 500-g fireworks rocket is moving with velocity $\vec{v} = 60\hat{\boldsymbol{j}}$ m/s
at the instant it explodes. If you were to add the momentum vectors of all its fragments just after the explosion, what would be the result?

A. $\vec{v} = 60\hat{\boldsymbol{j}}\ \text{kg}\cdot m/\text{s}$
B. $\vec{v} = 30\hat{\boldsymbol{j}}\ \text{kg}\cdot\text{m/s}$
C. $\vec{v} = 60000\hat{\boldsymbol{j}}\ \text{kg}\cdot\text{m/s}$
D. $\vec{v} = 30000\hat{\boldsymbol{j}}\ \text{kg}\cdot\text{m/s}$

Collision

http://www.specialised-imaging.com/

http://farm4.static.flickr.com



Collision

- Very short and violent interaction between objects
- Total momentum $\vec{P}$ is **always conserved** (assuming very short collision)
- Total kinetic energy K is not necessary conserved

  Elastic collision (K is conserved)

  Inelastic collision (K is not conserved)

  Totally Inelastic (two becomes one; or one becomes two)

Impulse

- During a collision, objects exchange momentum.

$$\Delta t = \text{duration of collision}$$
$$\Delta\vec{p} = \vec{F}_{ave}\Delta t \text{ (from Newtons 2nd law)}$$
$$\vec{F}_{ave} = \text{average force the object experiences during collision}$$
Warning: $\vec{p}$ **is for** ***an*** **object, not total!**
$$\textbf{Impulse: } \boxed{\vec{J} \equiv \vec{F}_{ave}\Delta t}$$
$$\Delta\vec{p} = \vec{J}$$

- "Pulling the table-cloth" trick works by minimizing the impulse (i.e. by minimizing the time).

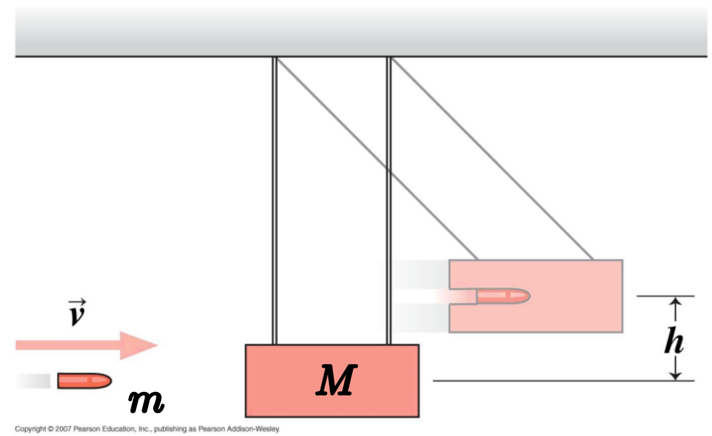
Quiz

Two skaters toss a basketball back and forth on frictionless ice. Which one of the following does not change?

A. The momentum of an individual skater
B. The momentum of the system consisting of one skater and the basketball
C. The momentum of the basketball
D. The momentum of the system consisting of both skaters and the basketball

Ballistic Pendulum (totally inelastic collision)

Ex. 9.10



- If block+bullet goes up by *h*, what was *v*?

1. Two becomes one: totally inelastic collision – only the total momentum is conserved.
2. Then, it becomes a “simple” pendulum: total mechanical energy is conserved.

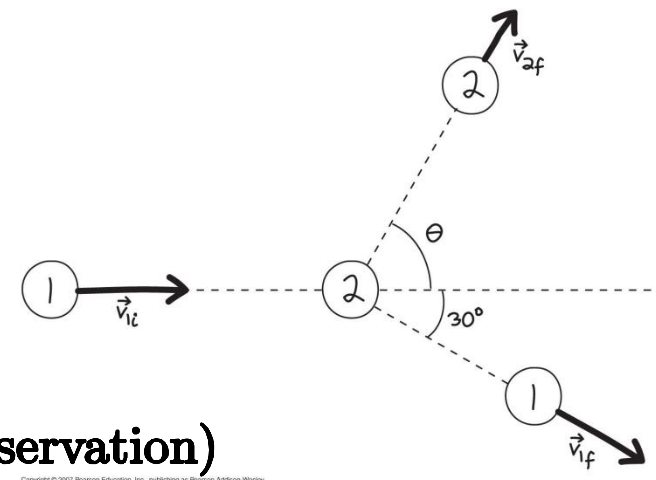
1. $mv = (M+m)v_f$
2. $(M+m)gh = \frac{1}{2}(M+m)v_f^2$

From 1, $v = \frac{M+m}{m} v_f$.
From 2, $v_f = \sqrt{2gh}$, and so
$v = \frac{M+m}{m}\sqrt{2gh}$

Croquet (Elastic Collision, 2D)

Ex. 9.12

- In an elastic collision, a ball strikes a stationary ball of the same mass, and comes out at 30 degrees. In what direction is the other ball moving?



$m\vec{v}_{1i} = m\vec{v}_{1f} + m\vec{v}_{2f}$ (momentum conservation)

$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$ (energy conservation)

In general it can be nasty to solve these two conservation equations.

In this case, it is simple since m cancels out. We get

$\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$ – (1)

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ – (2)

By taking dot product of (1) with itself on both sides,
and subtracting it from (2), we get

$\vec{v}_{1f} \cdot \vec{v}_{2f} = 0$ (i.e. the two outgoing velocities are orthogonal)

And so, $\theta = 60°$ in this case.