Power

During a small Δt, an object B moves by ~~displace~~ $\Delta\vec{r}_B$ while being pushed by an object A ("you") $\vec{F}_{AB}$

$$\text{Work} = \vec{F}_{AB}\cdot\Delta\vec{r}_B$$

Let us use symbol ΔW for this

$$\Delta W_{AB} = \vec{F}_{AB}\cdot\Delta\vec{r}_B$$

Power = work done <u>per unit time</u>

Average power

$$\overline{P}_{AB} = \frac{\Delta W_{AB}}{\Delta t} = \vec{F}_{AB}\cdot\frac{\Delta\vec{r}_B}{\Delta t} = \vec{F}_{AB}\cdot\overline{\vec{v}_B}$$

Average vel. of B

Instantaneous Power

$\Delta t \to 0$

$$P_{AB} = \frac{dW_{AB}}{dt} = \vec{F}_{AB}\cdot\vec{v}_B$$

instantaneous vel. of B

Units

Work = $N\cdot m \equiv J$ (joules)

Power = $J/\text{sec} \equiv W$ (watts)

$kW\cdot h$ = <u>unit of work</u> = $3.6\times10^{6}\ J$

Power → Work

$$W_{AB}(t_1 \to t_2) = \int_{t_1}^{t_2} dt\, P_{AB}$$

(Why? From Calculus. Follows from $P_{AB} = \frac{dW_{AB}}{dt}$)

§§ Work - Energy Theorem §§

Consider net force acting on obj B.
The ~~total~~ work done by net force (i.e. the work done by all objects that interact with B)

$$\Delta W_{net} = \vec{F}_{net} \cdot \Delta \vec{r}_B, \quad \vec{F}_{net} = \sum_A \vec{F}_{AB}$$

Make $\Delta \to d$ (infinitesimal → Appendix D)

$$dW_{net} = \vec{F}_{net} \cdot d\vec{r}_B$$

From now on, drop subscript B for simplicity.

Case 1 1 dimension

$$\vec{F}_{net} = m \frac{dv}{dt} \quad \text{(consider } \overset{\text{constant}}{\cancel{\text{fixed}}}\ m \text{ only)}$$

$$d\vec{r}_B = dx$$

$$\frac{dW_{net}}{dt} = m \frac{dv}{dt} \cdot \frac{dx}{dt} = m \frac{dv}{dt} \cdot v$$

Note that $\frac{d(v^2)}{dt} = 2v\frac{dv}{dt}$ and so $\frac{dv}{dt}v = \frac{1}{2}\frac{d(v^2)}{dt}$

(chain rule of diff. calculus)

$$\therefore \frac{dW_{net}}{dt} = \frac{1}{2} m \frac{d(v^2)}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$\boxed{\therefore W_{net}(t_1 \to t_2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$$

Case 2 **Any dimension** (still constant m)

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$

$$\frac{dW_{net}}{dt} = m\frac{d\vec{v}}{dt}\cdot\frac{d\vec{r}}{dt} = m\frac{d\vec{v}}{dt}\cdot\vec{v}$$

Note that

if dim > 2

$$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d}{dt}\left(v_x^2 + v_y^2 + \cdots\right) = 2\left(v_x\frac{dv_x}{dt} + v_y\frac{dv_y}{dt} + \cdots\right)$$

$$= 2\vec{v}\cdot\frac{d\vec{v}}{dt}$$ in any dimension!

$$\therefore \frac{dW_{net}}{dt} = \frac{1}{2}m\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$v \equiv |\vec{v}|$ magnitude of $\vec{v}$ speed

$$v^2 = v_x^2 + v_y^2 + \cdots = \vec{v}\cdot\vec{v}$$

So we get the same result!

$$\boxed{W_{net}(t_1 \to t_2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$$

Kinetic Energy $K \equiv \frac{1}{2}mv^2$

always positive or zero

Work-Energy Theorem

$$\boxed{W_{net}(t_1 \to t_2) = K_2 - K_1 = \Delta K}$$

Note: $\boxed{\text{unit of energy} = \text{unit of work}}$

J

kW·h

meaning: if an object receives a positive net work its kinetic energy goes up.

if receives a negative net work its kinetic energy goes down.