LECTURE VI

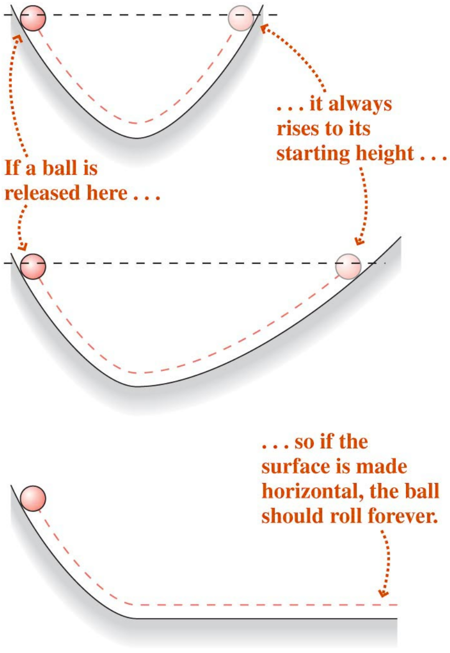
Force and Motion

VI.1. Newton's 1st and 2nd Laws

6.1. LAW. ***Inertia – Galileo's brilliant discovery***

A ball rolling on a smooth frictionless horizontal surface does not stop rolling. It will keep on rolling with the same velocity.

PROOF. Galileo was a superb experimentalist and an iconoclast. The [thought] experiment that led to this law is very simple but yet so powerful in its implications. The experiment is summarized in the following figure. In this experiment, we assume that, when the ball is on a horizontal surface, the gravity, acting perpendicular to the surface, cannot influence the motion. This discovery by Galileo Galilei can be summarized as "an object in motion will keep its motion as long as no force is applied." This way, he gave birth to modern science, Newton's laws and the principle of relativity, which became fully developed by Einstein later on. ☐



6.2. DEFINITION. **Body**

The word "body" refers to the object in motion, which we study. This can be a huge object such as a galaxy or a planet or a small object such as a baseball or a marble. Note that a body in classical mechanics is always a sizeable object. Nevertheless we often treat it as though it was a "point particle." Why is this? It is instructive to discuss an example. In order to describe the motion of the earth around the sun, to a good approximation, it is not necessary to take into consideration the spinning of the earth and the fact that the earth is a huge object from the human point of view. Instead, it suffices to assign only one position vector and one velocity vector and one acceleration vector to the earth, as though the earth was a "point particle" with zero size and no internal structure. Of course, if earth comes into close contact with another moving body, then its size and its internal structure will be of great importance. However, if sufficient time passes after such a collision, then the earth may again be treated as a "point particle." We won't be concerned with internal structures and internal motions until later in this course.

6.3. DEFINITION. **Force and net force.**

In Galileo's experiment, the reason that the ball keeps rolling on a horizontal surface is because nothing bothers it. Conversely, we define force roughly as that which causes the change of motion. We will come to a more precise definition, due to Newton, shortly. At this point, it should be realized that it is impossible to turn off all the forces. What can happen is that different forces can cancel one another and the net force is zero. Apparently, this is the case in Gelileo's experiment. We will analyze his experiment in more detail later on to see how this comes true.

The following law is due to Newton, but it was forseen by Galileo.

6.4. LAW. ***Newton's first law of motion***

A body in motion remains in uniform motion, and a body at rest remains at rest, unless disturbed by a nonzero net force.

6.5. DEFINITION. **Momentum** $\vec{p}$

$$\vec{p} \equiv m\vec{v}$$

where m is the mass of the body and $\vec{v}$ is the [instantaneous] velocity. Note that sometimes $\vec{p}$ is referred to as "linear" momentum as opposed to angular momentum, which we will learn later in this course.

6.6. LAW. ***Newton's second law of motion***

Let $\vec{F}$ be the net force acting on the body. Then,

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{vi.1}$$

or, for a body with a *constant* mass m,

$$\vec{F} = m\vec{a} \tag{vi.2}$$

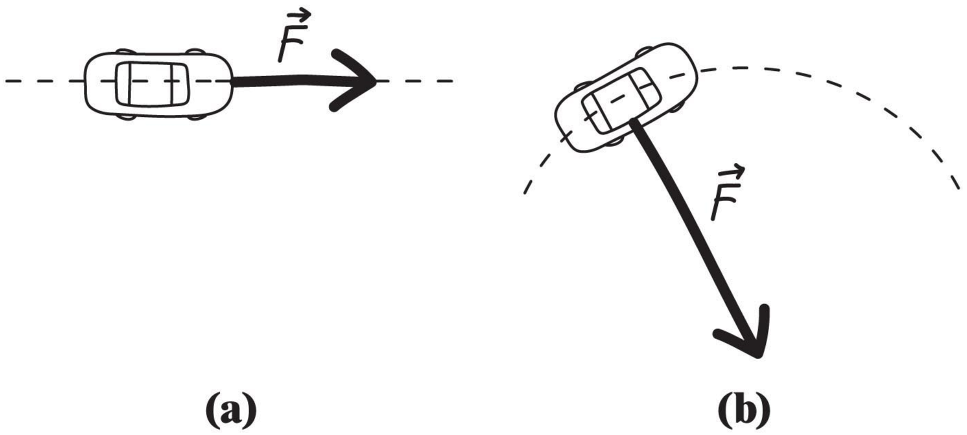
6.7. DEFINITION. **Unit of force: Newton (N)**

1 Newton $\equiv$ 1 kg m/s^2

6.8. OBSERVATION. ***Meaning of Newton's second law and inertia***

Newton's law is essentially the definition of force. You should understand it as *defining* force as "a vector quantity that causes the motion (momentum) to change." The momentum is $m\vec{v}$. From previous lectures, we are familiar with $\vec{v}$. But what is m? It is mass. It can be defined operationally as follows, while its absolute definition comes from higher level physics. Notice that for a constant mass $\vec{a} = \frac{\vec{F}}{m}$, i.e. the acceleration is inversely proportional to mass. That is, the more massive a body is, the more difficult it is to change its velocity. In this sense, m is often referred to as the "*inertial mass.*" By subjecting the same force and measuring the acceleration on two bodies, one of unknown mass and the other of known mass, the unknown mass can be obtained: $m_{unknown} = m_{known}\frac{a_{known}}{a_{unknown}}$.

6.9. EXAMPLE. Force on an accelerating car [Example 4.1 of textbook]



6.10. EXAMPLE. (a) m = 1200 kg, from rest to 20 m/s in 7.8 s. Force? Ans: $a = \frac{\Delta v}{\Delta t} = \frac{2.0\ \text{m/s}}{7.8\ \text{s}} = 0.26\ \text{m/s}^2$. $F = ma = 1200$ kg $\times 0.26\ \text{m/s}^2$ = 3.1 kN. (b) m = 1200 kg, $v = 20$ m/s, radius = 85 m. Circular motion. Force? Ans: $a = \frac{v^2}{r} = 20^2/85\ \text{m/s}^2$ = $4.7\ \text{m/s}^2$. $F = ma = 1200 \times 4.7$ N = 5.6 kN.

6.11. DEFINITION. **Inertial reference frame**

The reference frame was defined in Section IV.3 as a local (or locally convenient) coordinate system. An *inertial reference frame* is a reference frame in which Newton's second law is valid.

6.12. OBSERVATION. ***Non-inertial reference frame*** [optional; challenging]

It is often easier to know what something is by considering what something is not. The inertial reference frame is such an example. And, it is not an easy one to understand. But, here it goes. OK, first of all, we know for sure that on earth there is gravity pulling us down. However, suppose you were rich and/or famous enough to enjoy the adventure in a free-fall airplane (http://www.gozerog.com/). The coordinate system of that airplane is definitely not an inertial reference frame, since the gravity is *not felt* in that reference frame. Generally speaking an accelerating reference frame is not an inertial reference frame. Alas, then our local reference frame on our smallish spot on the surface of the earth is definitely not an inertial reference frame, since we know that the earth is spinning [any rotating object is accelerating as we saw in Section V.2]! Strictly speaking, this is correct, but the effect of the earth's spin such as the Corioli force is very small compared to other forces and thus to a good approximation it is fine to ignore the earth spin. [On a large scale "experiment" (like shooting cannons in a war) however, the non-inertial frame nature of the local coordinate system *must* be taken into account.] OK, then you say, the earth is not just spinning but it is also going around the sun! True. This actually makes the earth reference frame only an *effective* inertial frame. To understand why, let me go back to the free-fall airplane example. The question is this. You are in a free-fall airplane, and say you black out initially and wake up after a few seconds. You do not know where you are and what you are doing. You do experiments like Galileo or Newton did. Measure the law of inertia, Newton's second law, etc. What would you find? Would you still find that these laws are valid, assuming that you do not know where in the Universe you are and so you *do not know* that the earth is pulling you? Why?

VI.2. Gravity and Other Forces

When I was a high school student, I was told that there are four fundamental forces in Nature. Nowadays, that number is three. Soon, if the so-called “standard model” gets ironed out a little more, this may become two. And, if string theorists or quantum gravity theorists become successful, then ...! In any case, the forces in Nature include strong force (interaction between quarks), weak force (beta decay: $n \rightarrow p + e^{-} + \bar{\nu}_e$), electro-magnetic force, and gravitational force. The weak force and the electro-magnetic force, combined, is called the electro-weak force. For questions such as “what makes stars burn so bright?” or “where do protons/neutrons come from?,” weak and strong forces are important. However, the forces that we experience everyday are predominantly electromagnetic force or gravitational force. Of these, electromagnetic force is much stronger than gravity, and it underlies pretty much every thing such as tension force on a string, compression force of a chair, normal force, frictional force, etc. Gravity is famously a very feeble force, but at long distances it is the only force that survives, while other forces are “screened out” and so in planetary systems and larger systems, it is the only relevant force.

6.13. DEFINITION. **Weight**

Many scientific concepts become blurry and confusing in everyday lingo. In science classes, it is important to know the exact scientific definitions. Temperature and heat are well-known examples. (The following sentence is scientifically incorrect: “Today’s temperature will be warmer than yesterday.”) Mass and weight are also confusing terms. So, let us get them straight.

Mass is an **intrinsic** property of an object. Its origin is how many protons and neutrons a particular object is made of. It is measured in kg in the SI unit.

Weight ($\vec{w}$) is mass times the gravitational acceleration. $\vec{w} = m\vec{g}$. That is, weight is **force**.

People say “my weight is 150 lbs” or “I weigh 60 kg.” One has to be very careful in interpreting these statements scientifically. What this means is that on earth my weight is 150 lbs *times* g or 60 kg *times* g, where $g \approx 9.81$ m/s^2. So, if one weighs 60 kg, then that means that the earth is pulling her/him with force = 60 kg × 9.81 m/s^2 $= 6.0 \times 10^2$ N.

If we go to the Moon, then our weight becomes 6 times less, but our mass remains the same. What does this mean? If one brings a scale to the moon and measures her weight, then one would see the reading reduced by a factor of 1/6 on the Moon, while the number of protons that constitute her body does not change wherever she goes.

6.14. EXAMPLE. Mass and weight on Mars [Example 4.2 of textbook]

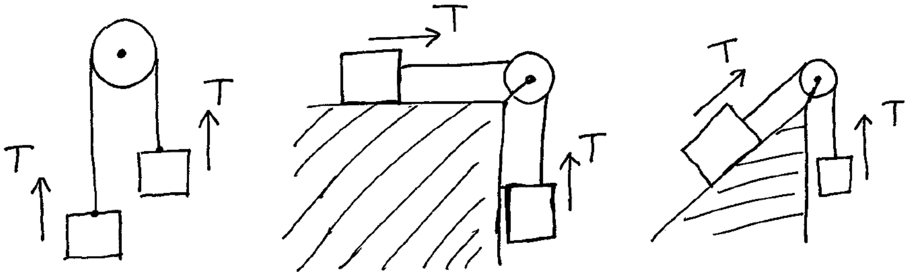
Rover Spirit weighs 1.8 kN on earth. $g_{Mars} = 3.74$ m/s^2 on Mars. Mass and weight on Mars? Ans: On earth: $m = 1.8$ kN / (9.81 m/s^2) = 180 kg (two sig-fig’s). On Mars: m remains the same = 180 kg (two sig-fig’s). The weight however changes $w = |\vec{w}| = mg_{Mars} = 180 \cdot 3.74$ N = 670 N (two sig-fig’s).

6.15. NOTE. **Apparent Weightlessness**

In a free-falling elevator, people do feel weightless, but according to the above definition weight is actually not zero? What is going on? The answer is that the reference frame of the free-falling elevator, or any other free-falling reference frame, is *not* an inertial frame. For the person inside the elevator, everything is falling with her, and so the weight is not felt.

6.16. Definition. **Tension Force**

Tension is force that a string or a rope exerts on a body connected to it. When a string is not accelerating, the tension forces at the two ends are equal in magnitude. However, the directions are in general different, and depend on the geometry of the string. If the string is accelerating, then the tension forces at the two ends can be different in magnitude, if the mass of the string is non-negligible. We will usually ignore the mass of a string, and then the magnitudes of the tension forces at the two ends of a string are always equal to each other, whether or not the string is accelerating. In this approximation, a string can be thought of as a force delivering mechanism, which conserves the magnitude of the force. The following figure shows different geometries in which the tension "delivers force."



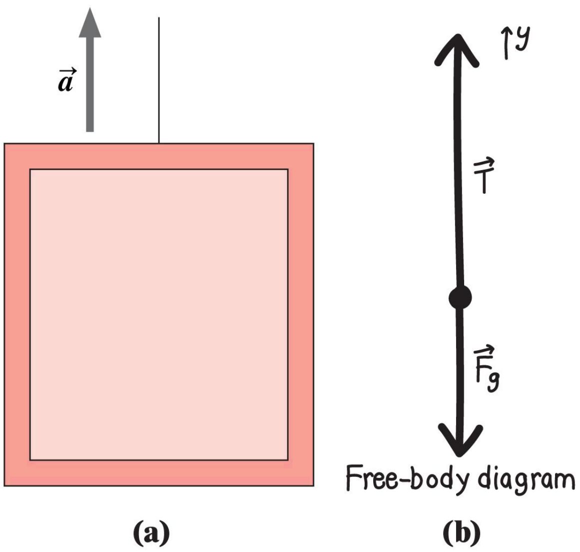
Tension "delivers" force. T is the same in magnitude at the two ends (even when the string is ~~in motion~~ accelerating if the string mass is negligible) while the directions can be different.

6.17. Definition. **Free body diagram and net force**

For a given body, a free body diagram is the diagram of all forces acting on that body. To get it: (1) Identify the object and all forces acting on it. (2) Reduce that body to a dot (center of mass). (3) Draw all forces with arrows, starting from the dot. The **net force** on the body is then the **vector sum of all forces**.

6.18. Example. Elevator. Free body diagram. [Example 4.3 of textbook]

Elevator is 740 kg. It is accelerating upwards at 1.1 m/s^2. Tension in the cable? Ans: $\vec{T} + \vec{F}_g = m\vec{a}$. Define the y axis as pointing up. Then $T - mg = ma$. $T = ma + mg = 740$ kg $\times (1.1 + 9.8)$ m/s^2 = 8.1 kN.


$\vec{a}$
$\uparrow y$
$\vec{T}$
$\vec{F}_g$
Free-body diagram
(a)
(b)