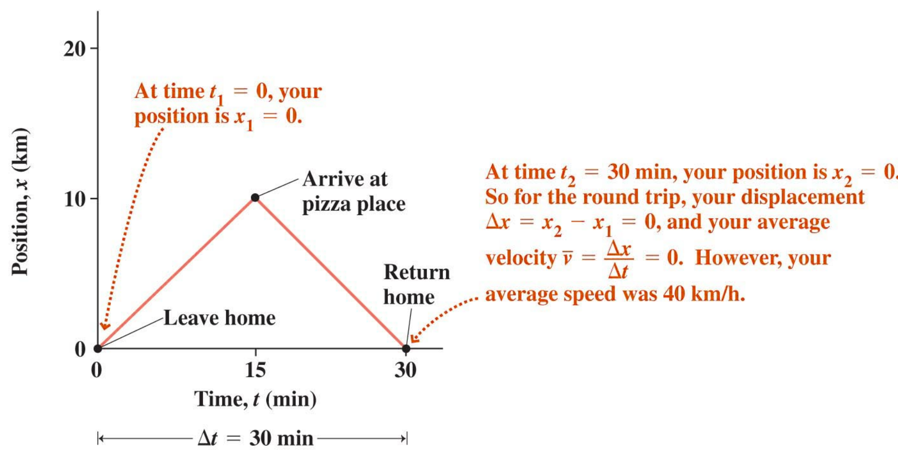
LECTURE II

Motion in One Dimension

II.1. Mechanics and Kinematics

Mechanics is the study of motion. Motion, or rather the *change* of motion as we shall see, is caused by something. Kinematics is the study of motion without specifying what that "something" is. We will deal with kinematics for a while. In this lecture, we cover kinematics in one dimension (1D), and in later lectures, kinematics in higher dimensions will be covered.

II.2. Average Speed and Average Velocity



From the above example the following important terms can be defined and illustrated. We consider the entire trip from home to pizza place to home.

Distance: The distance is the actual length of the motion. In this case, 10 km + 10 km = 20 km.

Displacement: The displacement is simply the difference between the final position and the initial position. For the round trip above, the displacement $\Delta x = 0$. (Here x is the symbol we use for the position. Δ is the conventional symbol for "the change in.")

Average Speed: The average speed is the distance divided by the duration. In the above example, 20 km / 30 min = 40 km/h.

Average Velocity: The average velocity is the displacement divided by the duration.[1]

$$\bar{v} \equiv \frac{\Delta x}{\Delta t}$$

(Here, $\bar{}$ is one of usual notations for "average" and $\equiv$ is used to mean "is defined as".) For the round trip above, it is 0 km / 30 min = 0 km/h.

[1] The textbook uses ν for velocity (or speed). Please note that in my note I will use a more widely used symbol v, which should be seen as interchangeable with ν of the textbook.

Note that if we concentrate on the first leg of the trip, then distance = displacement, and average speed = average velocity. For the second leg, it is more interesting. The full analysis of this trip is presented in Table II.1.

	t_i (min)	t_f (min)	x_i (km)	x_f (km)	Δt (min) $(t_f - t_i)$	Δx (km) $(x_f - x_i)$	distance (km)	average velocity $(\Delta x/\Delta t)$	average speed (distance / Δt)
first leg	0	15	0	10	15	10	10	40 km/h	40 km/h
second leg	15	30	10	0	15	**-10**	10	**-40 km/h**	40 km/h
round trip	0	30	0	0	30	0	20	0 km/h	40 km/h

TABLE II.1. Analysis of the above "pizza trip motion" for the first leg (home to pizza place), the second leg (back home from pizza place) and the entire trip. Note that the subscripts i and f are common notations for "initial" and "final," respectively.

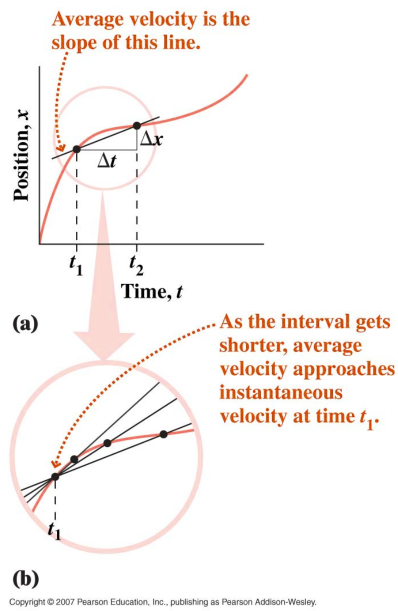
2.1. NOTE. **Vector and Scalar.** The above example, though simple, shows the difference between vector and scalar. Displacement and velocity have a sense of direction, as well as a sense of magnitude. This is why they can be negative (bold-face entries). Distance and speed on the other hand are characterized by magnitude alone. We say that displacement and velocity are *vector* quantities, while distance and speed are *scalar* quantities. We will discuss more about the distinction in later parts of this course. In this course, many basic quantities are vector quantities, and so we will review vectors more in depth in a later lecture.

II.3. Instantaneous Velocity and Instantaneous Speed

2.2. DEFINITION. **Instantaneous Velocity** in 1D, $v(t) \equiv \lim_{\Delta t \to 0} \bar{v}$.

$$v(t) \equiv \lim_{\Delta t \to 0} \frac{\Delta x(t)}{\Delta t} \equiv \frac{dx(t)}{dt} \equiv \dot{x} \tag{ii.1}$$

The graphical meaning of $v(t)$ in 1D is the slope of the (t, x) graph. The following figure illustrates the average velocity in the time interval (t_1, t_2) and the instantaneous velocity at t_1.



2.3. DEFINITION. **Instantaneous speed** in 1D $\equiv |v(t)|$.

II.4. Acceleration

2.4. DEFINITION. **Average Acceleration**

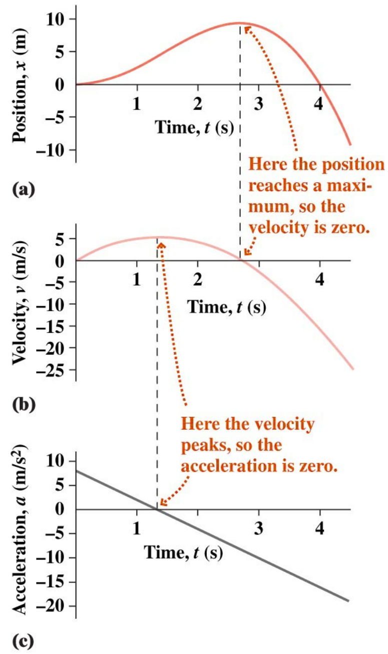
$$\bar{a} \equiv \frac{\Delta v}{\Delta t}$$

2.5. DEFINITION. **Instantaneous Acceleration**

$$a(t) \equiv \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta v(t)}{\Delta t} \equiv \frac{dv(t)}{dt} \equiv \dot{v} \quad \text{(ii.2)}$$

Note: The acceleration is a vector also, as we will prove rigorously later. At the moment, the vector nature of the acceleration can be understood this way: since what a is to v is what v is to x, and x and v are vectors, a must be a vector also.

The graphical meaning of the instantaneous acceleration is the slope of the graph (t, v). Note that, from this definition and Def. 2.2, it follows that $a(t) = \frac{d^2x}{dt^2}$. Thus, $a(t)$ says how fast the slope of the graph (t, x) is changing. [**Curvature part removed. $a(t)$ is related to the curvature of x, but not exactly it.**]


Position, x (m)
10
5
0
−5
−10
1
2
3
4
Time, t (s)
(a)
Here the position reaches a maxi-mum, so the velocity is zero.
Velocity, v (m/s)
5
0
−5
−10
−15
−20
−25
Time, t (s)
(b)
Here the velocity peaks, so the acceleration is zero.
Acceleration, a (m/s²)
Time, t (s)
(c)