LECTURE I

Introduction, Units, and Significant Figures

I.1. What is physics, or this course, all about?

Physics is a discipline of natural philosophy. It aims to establish physical laws by which diverse phenomena can be understood ("classified") in a neat way. Over the human history, it has served us well in establishing knowledge about how celestial bodies move, how vehicles and tools operate, and how atomic and sub-atomic devices work.

In Physics, we strive to establish the simplest laws that covers the most diverse phenomena. Let me point out two things that are direct consequences of this last statement.

1.1. OBSERVATION. ***The joy of figuring things out***

Physicists do not tend to memorize a lot of things, since it is almost always the best strategy to understand the basic principles and learn to apply them "on the fly" to various problems. During this course, I encourage you to acquire this "on the fly" mind-set, if you do not have it already. How does one acquire it? How does one know what the basic principles to "memorize" are? Perhaps there is no one answer for everyone, but I think a good general answer is "through lots of work, doing problems with pencil and paper." Because physics questions are almost never the "fill-in-the-blank" type that test whether something is in your memory or not, often they look difficult and intimidating at first. By putting one's energy, time, and intellect to work to understand, re-formulate and solve problems after problems, one gradually learns how to approach physics problems and what the basic principles are. And the initial fear erodes. The basic principles are mostly so simple (e.g. Newton's laws, or momentum conservation) that to say that one "memorizes" them is an overstatement. They just stick to you as you put your labor into doing problems. So doing lots of problems is an essential part of learning physics. (It is no wonder then that you hear people say "physicists are problem solvers" often in relation to job market.) [For more strategies of learning physics, reading Section 1.4 of the text may be helpful.]

1.2. OBSERVATION. ***Physics is basic and in demand.***

Physics tends to underlie a wide range of disciplines – chemistry, biology, engineering. A well-known consequence is that it is the driving force of our information industry. Undoubtedly it is also the underlying driving force for the up and coming alternative energy industry. Of course, this does NOT mean that other disciplines are simply applied physics. Each discipline has its beautiful internal structure, which springs up largely on its own. However, it does mean that learning physics is a required step for many other disciplines. Unfortunately, mind sets best suited for learning these other disciplines tend to be a little different from the mind set best suited for learning physics. There is nothing wrong with this fact or people having different mind sets. However, what it means is that, *in this course* every student is expected to learn to acquire the physics mind set. So, we go back to the above discussion. Also, using common sense wisdom (e.g. ask [help] and it shall be given; the more, the merrier – your classmates are your friends/co-adventurers, talk your problems out) should be very helpful. Of course, we should never forget to retain healthy mutual respects of different disciplines.

In this course, I will help you learn the essential principles of classical mechanics at an introductory level. This is a beautiful subject. Some of you might think that it is kind of boring, since we are so used to these concepts ("force," "energy," ...) or at least to hearing about them. This is the science that was developed 300 hundred years ago! This was roughly how I felt when I was sitting in class as a college freshman, so I kind of know how some of you might feel. However, please rest assured that these subjects are by themselves still such wonderful and ever intriguing subjects to scientists and engineers. Science may be many things, but it is never boring. It is always true, one may assert, that we can relate to the frontier of science what is happening right under your nose, or inside you for that matter.

OK, what subject matters are we going to cover in this course? Simply put, we will cover Newton's laws and conservation laws. Newton's laws are breath-taking beauties. Conservation laws refer to the laws concerning energy, momentum, and angular momentum *conservations*. Since this is the first lecture, let me make some general remark about conservation laws and why they are important. Conservation and symmetry are two sides of a coin, and so conservation laws can be called symmetry laws. It is Newton himself who is credited to have devised the concept of "symmetry" – he wondered, as legend puts it, whether the principle that governs the falling apple is applicable to celestial bodies in the heaven as well. Galileo Galilei also contributed to the germination of such an idea, through his famous thought experiment of a ball rolling down a hill. It is Newton's breathtaking success that he could formulate laws that beget the symmetry principles that he intuitively started with – conservation of momentum (and others). In Newtonian mechanics, the subject of this course, the conservation laws were thus thought to be derived from the mechanical laws. However, let me mention that, since Einstein's time conservation laws have gained apparently greater importance than mechanical laws. This is perhaps why you hear so much about "symmetries" when people talk about science (like LHC).

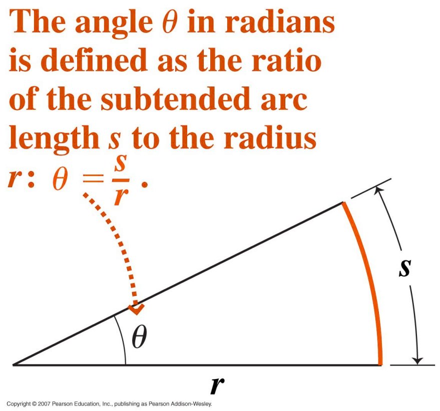
I.2. Units, Dimensions, and Significant Figures

Units are extremely important for physics, and a basic course such as this should start with discussion of units. Indeed, our excellent text does just that.

1.3. DEFINITION. **The SI unit system**

The standard unit system used is called the **SI unit**. I find the old name for it more informative for the purpose of this course; the **MKS unit**. M is for meter (m), K for kilo-gram (kg), and S for second (s). These three base units, when combined, form units for any quantities related to motions. The SI unit system has other base units (ampere, kelvin, candela, mole), but we won't be using them in this course. A sister unit system of the SI system the CGS unit system, in which centi-meter (C), gram (G), and S (second) are used as base units. MKS (i.e. SI) and CGS systems collectively belong in the "metric system." Many regional non-metric units exist, the most notable being the English unit system (inch, pound, etc.) widely used in this country. **We will stick to the SI unit system**, as this is the standard practice at this level of physics. However, to make connection to our senses of everyday objects and phenomena, it would be important to be able to convert from the English unit to the SI unit and back.

One quantity worth emphasizing is angle. The SI unit of angle is "radian," and is related to degrees by 2π radian = 360 degrees. You can remember it by reasoning that the circumference of a unit radius circle is 2π. Indeed, the definition of radian is the ratio of the arc length and the radius as in Figure 1.2 of the text (copied below), namely the angle in radian is the length of the arc for a unit radius.



1.4. Definition. **Unit Suffixes**

Given the basic unit such as m, kg, and s, prefixes are used to describe certain multiples or fractions. See table 1.1 of the text (copied below). It is important to remember prefixes such as **centi (c) = 10^{-2}**, milli (m) = 10^{-3}, micro (μ) = 10^{-6}, kilo (k) = 10^{3}, mega (M) = 10^{6}, giga (G) = 10^{9}. Please do not confuse m (milli) and M (mega). For instance, "Watt (W)" is a derived unit for power, and mW means milli-Watts (10^{-3} Watts), while MW means mega-Watts (10^{6} Watts) – a world of difference! **Note also that all prefixed powers here, if greater than 100 or less than 0.01, are powers of a thousand,** 10^{3n}**,** $n = \pm 1, \pm 2, \ldots$**.**

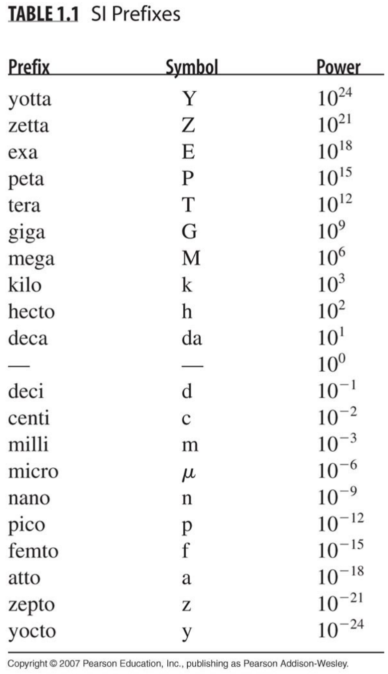


TABLE 1.1 SI Prefixes

Prefix	Symbol	Power
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
—	—	10^{0}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

1.5. Example. **Unit conversion**

The speed of a car is 60 miles per hour (or 6.0E1 miles per hour). What is the speed in the SI unit? One mile is 1609 meters. And one hour is 3600 seconds. So,

$$60\,\frac{\text{mile}}{\text{hour}} = 60 \times \frac{1609\,\text{m}}{3600\,\text{s}} = 27\,\frac{\text{m}}{\text{s}}$$

i.e., 27 meters per second. As you can see from this example, the unit conversion proceeds following the normal multiplication and division rules, where "per" means divided by. Here, we assume that the number of significant digits is 2 [we will discuss about the significant digits shortly].

Here is a table, in three significant digits and then in one significant digits (for use in a quick but useful estimations – see Definition 1.15), showing how to go from metric units to English units.

Metric system	English system	English system	Comments
1 m	1.09 yd	~ 1 yd	1 yd = 3 ft
1 kg	2.20 lbs	~ 2 lbs	5 kg ≈ 11 lbs (read a supermarket scale)
1 m	3.28 ft	~ 3 ft	1 ft ≈ 30 cm
1 cm	0.394 in	~ 0.4 in	1 in = 2.54 cm
1 km	0.621 mile	~ 0.6 mile	80 km ≈ 50 mile (read your speedometer)
1 m/s	2.24 mph	~ 2 mph	

TABLE I.1. Conversion of some metric units to commonly used English units

1.6. DEFINITION. **Dimension**

The concept of **dimension** is intimately connected with the concept of units, and it is very much worth knowing. We talk of our three-*dimensional* (3D) world. The implication is that our world is described by three numbers, for instance the coordinates x, y, z, each representing an independent direction. Consider a cube of side length l. The volume is $V = l^3$, and has the dimension of length^3. One can consider other objects such as a sphere, for which $V = \frac{4\pi}{3}r^3$ (r=radius), or a cylinder, for which $V = \pi r^2 h$ (r=radius, h=height). While these expressions differ from each other, the fact that $V \propto$length^3, or the fact that the unit of V is m^3 in the SI system, is common to these objects. Thus, we say that the dimension of the volume is length-cubed. Of course, the underlying reason for this is the three coordinates x, y, z. Special symbols are reserved to denote the dimension. In this note, it suffices to know L (length), M (mass) and T (time). We can summarize our finding regarding the volume as

$$[\text{volume}] = L^3$$

where the angular bracket [] means "the dimension of." The exponent, 3, of L here is what we mean when we say that we live in a 3D world. If we lived in a 2D world, the standard "volume" would be the area, for which $[\text{area}] = L^2$.

Generalizing this, one talks about physical dimension of any general quantity. For instance, the velocity has the unit of m/s. We write

$$[\text{velocity}] = LT^{-1}$$

This equation reads as "the dimension of velocity is length (L) divided by time (T)." Another example is momentum which, as we will see later, has units of kg m/s (or kg-m/s). This means

$$[\text{momentum}] = LMT^{-1}$$

which reads as "the dimension of momentum is length (L) times mass (M) divided by (T). Generally powers larger than 1 or smaller than -1 can be involved. For instance, the acceleration is, dimension-wise, velocity divided by time and so

$$[\text{acceleration}] \;=\; LT^{-1}/T = LT^{-2}$$

All quantities of our concern in this course have dimensions that can be expressed in terms of L, M, T only, not involving other units such as temperature etc. Note that some quantities have "no dimensions" in the sense all exponents are zero. A good example is the angle.

$$[\text{angle}] \;=\; L^0 M^0 T^0$$

Such quantities are said to be "dimension-less."

1.7. FACT. ***Some basics about dimension***

(1) A physics equation A = B necessarily means that [A] = [B]. If [A] = [B], then A and B can be added or subtracted.
(2) Two physical quantities of different dimensions cannot be compared, nor can they be added or subtracted. They are "apple and orange."
(3) **Same units mean same dimensions. Same dimensions mean possibly same units.**

1.8. OBSERVATION. **WHAT TO CHECK WHEN IN DOUBT I**

On solving a problem, the first thing that a physicists does after obtaining a solution is to check the dimension. If the calculation turns out to be long and winding, the dimension check is done at intermediate steps, to guard against any silly mistakes, which of course happen to everyone ["Experts" are those who learned from lots and lots of mistakes]. For instance, if the question asks a momentum value, and if one gets an answer that is 3.14 m^2/s or an expression such as $\sqrt{10}m$ (m=mass), then something went wrong for sure during the calculation.

1.9. FACT. ***Dimensions of common expressions***

Dimension-wise, Qq and $\int Qdq$ have the same dimensions where Q and q are arbitrary physical variables/quantities. Also, $\frac{dQ}{dq}$ and Q/q have the same dimensions. Lastly, in expressions, such as $\exp(x)$, $\sin(x)$ etc., involving analytic transcendental functions, x should be dimensionless.

PROOF. The first two statements are left for readers to prove (starting from definitions of integral and differentiation). For the last statement, note that an analytic transcendental function of x can be expressed as an infinite power series, $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, where $a_0, a_1, ...$ are dimensionless, since they are just numbers [for instance, $\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$]. Thus, it follows from the last of Fact 1.7 that 1, x, x^2, ... have the same dimension. Thus, $[x] = [1]$. That is, x is dimension-less. □

1.10. DEFINITION. **Significant Figures/Digits**

Suppose you weigh yourself on a scale in the morning and the scale displays "149" pounds. What does this mean? Assuming that the scale is calibrated pretty well, this means that your weight w satisfies $148.5 \leq w < 149.5$ pounds. Here, we say that there are three significant figures/digits. Now suppose that one measures the diameter of one's hair and comes up with an estimate that it is about 50 μm, in the sense that it is between 45 and 55 μm. If this is the case, the number of significant figures is only one as we cannot say for sure that the estimate is definitely not 51 or 52. However, just writing "50 μm" is ambiguous, since a reader may wonder whether the trailing 0 is significant. Instead, in this case, one better write **5×10^1 μm** or 5×10^{-5} m. This unambiguous notation is called the "scientific notation."

1.11. DEFINITION. **Scientific Notation**

Any real number can be expressed as

$$r \times 10^n$$

where r is a real number, and n is an integer. In the normalized scientific notation, we restrict that $1 \leq |r| < 10$. In this course, we use "scientific notation" as synonymous to "normalized scientific notation" [except when the number is zero, in which case the normalization is not possible]. Computer programs and calculators use the following notation

$$r\mathrm{E}n \equiv r \times 10^n$$

and I will sometimes use this notation to save typing. An alternative notation, ren, however will not be used at all since it can be confusing with the exponential function.

1.12. DEFINITION. **Number of significant figures/digits**

It is the number of digits in r of the above [normalized] scientific notation.

Note: According to this definition, if any number in the non-scientific notation starts with 0s, then those 0s are not significant. For instance, 0.0004 has only 1 significant figure, since its scientific notation is 4×10^{-4}. This is consistent with our common sense. The four 0s in 0.0004 are just place-holders. On the other hand, if one writes -0.000500, then the 2 zeros at the end should be interpreted as significant as there is no other reason for those to be there (however, -5.00×10^{-4} would be a more recommended way to express the same meaning). 0s at the end of integers such as "-40" or "500" are totally ambiguous as to their (in)significance, and therefore it is not possible to say exactly how many significant figures are in these two numbers [cf. Observation 1.16].

It is advised that, when you use your calculator for this course and the lab course, you set it to the scientific mode that displays the numbers in the scientific notation.

1.13. FACT. ***Rules of thumb regarding significant figures***

(1) When adding or subtracting numbers, keep the smaller number of digits to the right side of the decimal point for the two operands (after you express them with the same exponent n, which should be the ~~smaller~~ larger exponent of the two). Note that this is before expressing the result in the scientific notation. (cf. the last "addition, subtraction" example below.)

(2) When multiplying or dividing numbers, keep the smaller number of the significant digits of the two operands.

(3) For a more complex functions such as square root, sin, cos, etc., ~~follow 2.~~ the # of sig-figs remain unchanged

(4) For multi-step calculations, keep at least one more significant digit in the intermediate steps than is required at the end in order to prevent rounding error accumulation.

If you are curious where these rules comes from, these are rules of thumb that follow from the error propagation theory, which should be consulted ultimately if ambiguities and questions arise. For further information, lab courses should provide more details. There are also resources in the web (wikipedia). Also feel free to ask me outside class.

1.14. EXAMPLE. **Significant figures and arithmetic**

Addition, subtraction: [largest error bar wins] [Note: one more example added at the end; always remember "largest error bar wins"]

$$\begin{aligned} 3.75\text{E}6 + 5.2\text{E}5 &= 3.75\text{E}6 + 0.52\text{E}6 = 4.27\text{E}6 \\ 3.7\text{E}6 + 5.2\text{E}5 &= 3.7\text{E}6 + 0.52\text{E}6 = 4.2\text{E}6 \\ 3.7\text{E}6 - 5.256\text{E}5 &= 3.7\text{E}6 - 0.5256\text{E}6 = 3.2\text{E}6 \\ 3.789\text{E}6 - 5.1\text{E}3 &= 3.789\text{E}6 - 0.0051\text{E}6 = 3.784\text{E}6 \\ 1.349 - 1.2 &= 0.149 = 0.1 \text{ (can only keep 1 digit after ".")} = 1\text{E-}1 \end{aligned}$$

Multiplication, division: [keep the fewest number of significant figures, at the end of calculation]

$$\begin{aligned} 3.0\text{E}8 \times 2.16\text{E-}10 &= (3.0 \times 2.16) \times 10^{8-10} = 6.5\text{E-}2 \\ 3.0\text{E}8/2.16\text{E-}10 &= (3.0/2.16) \times 10^{8+10} = 1.4\text{E}18 \end{aligned}$$

More complex: [like multiplication, division]

$$\begin{aligned} \sqrt{(3.61\text{E}4)^3} &= \sqrt{(3.61)^3 \times 10^{4\times 3}} = (47.05 \times 10^{12})^{1/2} \\ &= (47.05)^{1/2} \times 10^{12\times 1/2} = 6.86 \times 10^6 \end{aligned}$$

Note that in intermediate steps, one can keep as many significant figures as possible, but one should always remember to keep the right number of significant figures at end. The reason to keep more [one more is enough, actually] significant figures at intermediate steps is because, if one keeps too few significant figures at intermediate steps, rounding errors can make error bars larger than they really are.

1.15. DEFINITION. **Order of magnitude estimation**

This crude "back of the envelope" estimation method is often a very useful way to quickly assess certain scientific statements or answers to physics questions. It is an estimation rounded to the nearest power of ten, while often one significant digit is kept, with an error bar larger than strictly implied by that digit. Physicists notation for this estimation is usually $\sim$. For instance, Example 1.4 of the text estimates that the number of cells in a brain $\sim 10^{12}$ cells. This is enough to choose a correct answer for a multiple choice situation where, say, the choice has boiled down to between (a) 10^{11} and (b) 10^{15}.

1.16. OBSERVATION. ***Human nature***

By human nature, even scientists and physics textbook writers use expressions such as "100 feet" when what they really mean is "1.00×10^2 feet," "1.0×10^2 feet" or "1×10^2 feet." The rule of thumb that we will use in this course would be that an ambiguous expression such as "100 feet" given without any further specification means the most accurate one "1.00×10^2 feet."

1.17. OBSERVATION. **WHAT TO CHECK WHEN IN DOUBT II**

The second thing that a physicist checks when a solution is obtained is the order of magnitude sensibility. In this course, we will mostly talk about velocity, force and weight, etc. To establish a quick sense of the order of the magnitude, the conversion factors, presented in Table *I.1* with only one significant digit (3rd column), should be useful. For instance, if one is asked a question about the speed of a car, and if one obtains an answer v =20 m/s, then it means

$$v = 20 \text{ m/s} \sim 20 \times 0.6\text{E-3}\,\text{mile} \times 3.6\text{E3/hour} \sim 40\,\text{mile/hour}$$

which is reasonable. On the other hand, an answer such as 200 m/s would be seen as unreasonable, and requires a serious double-checking.

1.18. SUMMARY. **Know your quantities.**

If you are well-aware of **units**, **dimensions**, and **order of magnitudes**, you will have a great joy of figuring things out.

1.19. NOTE. **Fine print about error bars (uncertainty)**

In your lab work, you should always use the correctly determined error bars explicitly! Specifying the error bar, any result should be reported in the format of

$$\textit{value} \pm \textit{error}$$

For example, the gravitational constant G is known as

$$6.67259 \times 10^{-11} \pm 8.5 \times 10^{-\overset{15}{\cancel{17}}}\,\text{N} \cdot \text{m}^2/\text{kg}^2$$

or in other words

$$6.67259 \times 10^{-11} \pm 0.000\cancel{00}85 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$$

A powerful short-hand “parenthesis” notation is more widely used to mean the same thing (as you can understand why):

$$6.67259(85) \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$$

In this lecture course, we will never specify the error bars explicitly in problems (although in *real* problems you should), basically for the simplicity and convenience of teaching and learning. When error bars are not reported explicitly then the convention is that *the order of magnitude* for error bars is implied just by looking at how many significant figures are used. This is the underlying assumption of what is said in the above discussions of significant figures. So if one says that the weight (or, actually, *mass*) is “1.49×10^2” pounds, without any other specification, then it is implied that the error bar is roughly ±0.5 pounds. The actual error bar, if measured more carefully, may be somewhat larger or smaller than that but it should not be ten times larger or smaller.