Physics 6A **Introduction to Physics** **Practice Exam**

Final Exam

- **Closed book. No notes.**
- **Calculators with cleared memory are okay.**
- ***Show your work on all calculations.***

Equations for motion in one dimension with constant linear or angular acceleration:

$x = x_0 + v_0 t + \frac{1}{2}at^2$ $x = x_0 + \frac{1}{2}(v_0 + v)\cdot t$ $v = v_0 + at$ $v^2 = v_0^2 + 2a\cdot(x - x_0)$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha\cdot t^2$ $\theta = \theta_0 + \frac{1}{2}(\omega + \omega_0)\cdot t$ $\omega = \omega_0 + \alpha\cdot t$ $\omega^2 = \omega_0^2 + 2\alpha\cdot(\theta - \theta_0)$

Newton's second law: $\vec{a} = \frac{1}{m}\cdot\vec{F}_{\text{net}}$ or $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$ $\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$ or $\tau = I\alpha$

Torque: $\vec{\tau} = \vec{r}\times\vec{F}$

Moment of Inertia: $I = \sum_i m_i r_i^2$

Solid cylinder: $I = \frac{1}{2}MR^2$, solid sphere: $I = \frac{2}{5}MR^2$, stick of length ℓ about one end: $I = \frac{1}{3}M\ell^2$

Friction: $f_k = \mu_k F_N$ $f_s \le \mu_s F_N$

Circular motion: $a_{\text{rad}} = \frac{v^2}{r} = r\omega^2$ $a_{\tan} = r\alpha$ $v = r\omega$ $s = r\theta$

Work: $dW = \vec{F}\cdot d\vec{s}$ $F_x(x) = -\frac{dU}{dx}$

Mechanical Energy: $E = U + K$ $K = \frac{1}{2}mv^2$ or $K = \frac{1}{2}I\omega^2$ Power: $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$

Momentum: $\vec{p} = m\vec{v}$ Angular momentum: $\vec{L} = \vec{r}\times\vec{p}$ or $L = I\omega$ Impulse: $\vec{J} = \Delta\vec{p} = \vec{F}_{\text{avg}}\cdot\Delta t$

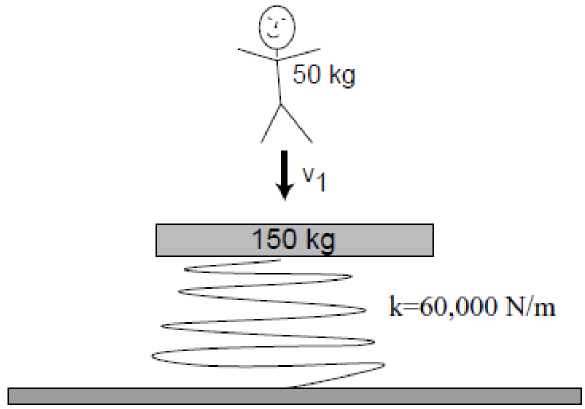
Mass on spring: $F = -kx$ $U = \frac{1}{2}kx^2$

Gravity: $g = 9.8\ \text{m/s}^2$, $G = 6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$ $U = mgh$ or $U = -G\frac{m_1 m_2}{r}$ $F = G\frac{m_1 m_2}{r^2}$

Simple harmonic oscillator, spring: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ pendulum: $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$ $\omega = 2\pi f$ and $T = \frac{1}{f}$

Simple harmonic oscillator: $x(t) = A\cos(\omega\cdot t + \phi)$ $\frac{d^2x}{dt^2} + \omega^2 x = 0$

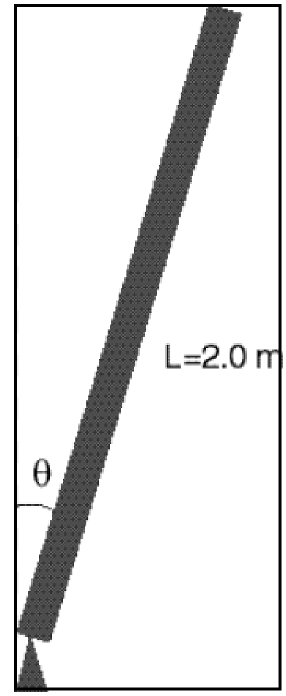
1. A 50 kg woman falls from a ladder down onto a 150 kg platform, which rests in equilibrium upon a spring of negligible mass and spring constant k=60,000 N/m. After landing upon the platform, the spring is observed to compress by an amount $x_c = 0.10$ m.



a) What was the speed, v_2, of the woman plus platform immediately after she landed but before the spring began compressing?

b) What was the speed, v_1, of the woman immediately before she landed? (Hint: it was a completely inelastic collision.)

2. One end of a uniform stick of length L is placed upon a point such that it is leaning at an angle θ from the vertical, as indicated in the figure. It is then released and allowed to topple over.

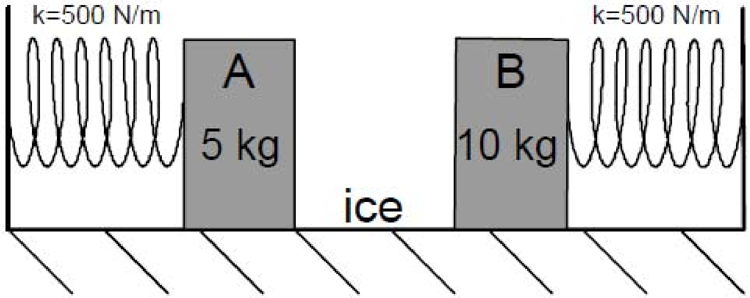


a) Derive an expression for the torque exerted by gravity on the bar, expressed in terms of m, g, θ, and L. Take the axis of rotation to be at the lower end of the bar.

b) Derive an expression for the angular acceleration α of the stick about the point at the moment it is released, in terms of g, θ, and L.

c) If L=2.0 m, what is the angular velocity ω of the rod (rotating about the point) when it is horizontal (θ=90°), assuming that it started out very close to vertical (θ=0) and with zero initial angular velocity. Assume that it does not slip off of the point, which acts as a frictionless pivot. (Hint: do not try to do this by using formulas for motion with constant angular acceleration, as the acceleration derived in part (a) varies with the angle. Making use of a conservation law would be more appropriate.)

3. Block A, of 5.0 kg mass, and Block B, of 10 kg mass, are resting on a frictionless surface. Each has a massless spring of spring constant k=500 N/m between it and the wall, as shown in the figure below. The blocks are not connected to the springs, which are initially in equilibrium. Block A is displaced to the left by 0.25 m, compressing the spring, and then released. It slides to the right, striking Block B in a completely inelastic collision, after which the two blocks move together.



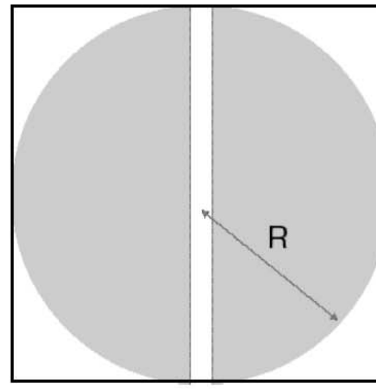
a) What is the speed of Block A just before the collision?

b) What is the speed of the two blocks immediately after the collision, before the second spring has compressed any significant amount?

c) What is the maximum distance that Block B moves to the right?

d) How much time passes between the collision and the moment when Block B first returns to its initial position?

4. A hypothetical planet is a uniform sphere of mass M and radius R, except that it has a narrow, straight tunnel bored from a point on the surface all the way through the center and out the opposite side of the sphere. A rock of mass m is dropped into the tunnel from the surface at time $t=0$. It can be shown from Newton's law of gravity (Example 12-9 in the textbook) that the gravitational force on the rock while it falls through the tunnel is directed toward the center of the planet and has a magnitude proportional to the distance from the center:



$$F = \left(G\frac{Mm}{R^3} \right) \cdot r$$

a) Assuming that the tunnel has no air inside, then the rock will travel back and forth from one side of the planet to another in a harmonic oscillation, with $r(t) = R\cos(\omega \cdot t)$. Explain clearly why this is so, based on Newton's laws and the above information.

b) Assuming that the planet has the mass and size of Earth ($M=6.0\times 10^{24}$ kg and $R=6.4\times 10^{6}$ m), how long, in seconds, will it take for the rock to go from one side of the planet to the other and return?

5. A super powerful spring gun is designed to launch a 2.0 kg ball into outer space from the surface of the moon. Given that the mass of the moon is 7.4×10^{22} kg and its radius is 1.7×10^{6} m, and assuming that the spring constant is k=800,000 N/m, how far must the spring be compressed in order for the ball to escape from the moon (that is, to be launched to infinite height)?