APPENDIX C

Significant Figures (Recap)

Significant figures appear to be a perplexing subject to many students. I definitely do not blame you! Here goes the clearest explanation that I can give on tricky aspects of it. [You should still read Lecture I for general discussions.]

First, what does "significant" in "significant figure" really mean? Right here seems to lie the source of confusion. Of course, in a sense, all digits that we use to represent numbers are significant (otherwise, we would not use them!). So "significant" in "significant figure" must mean something else. What it actually means is "used to distinguish between numbers of the same order of magnitude." Put another way, if a digit is used to *merely* express the order of magnitude, then that digit is not significant. Accordingly only 0 can be insignificant. In the [normalized] scientific notation, $r \times 10^n$, the integer n is the order of magnitude, and $1 \leq r < 10$. So, the number of digits that are used to express r is the number of significant figures (as in Def. 1.12)

Examples: $0.001 = 1 \times 10^{-3}$ (# of sig fig = 1; none of three zeros in 0.001 are significant; why? because if any of those 0's are changed to a non-zero digit, then the order of magnitude changes) $102.3 = 1.023 \times 10^2$ (# of sig fig = 4; 0 in 102.3 is significant; why? because if I change it to any non-zero digit, the order of magnitude does *not* change) 100 (# of sig fig = ambiguous; why? well, the problem here is that it is not clear whether 0's here are written to *merely* express the order of magnitude or not; In other words, it is not clear whether the scientific notation should be 1×10^2, 1.0×10^2 or 1.00×10^2) 4.00 or 0.300 (# of sig fig = 3 in either case; why? It is clear that these two trailing 0s are not there for the purpose of expressing the order of magnitude; In scientific notations these numbers must be written as 4.00×10^0 and 3.00×10^{-1}). Q: How many significant figures are in 0.0001004500?

Second, the concept of significant figures is closely related to the underlying error bar. This is common-sensical and examples should suffice.

Examples: The rounding rule is the key notion here. If a person writes 0.001, then by rounding rule, that number must mean a number between (inclusively) 0.0005 and 0.001499999.... So, as you can see, if $x = 0.001$, then, when error bar (i.e. uncertainty) is not explicitly given, the scientific meaning is that $x = 0.001 \pm 0.0005$. If a person writes $x =$0.00100, however, it should be interpreted that the trailing 0's are significant, and the meaning is that $x = 0.00100 \pm 0.000005$. Again notice that if one says $x = 10$ (ambiguous sig fig's) then it is not clear what the error really is.

Third, in addition or subtraction the result should have the same error bar of the larger of the two error bars associated with the two operands. Why so? Consider an example of a bee in a room. We ask the question "what is the bee's vertical position?" We answer by measuring it. Let us say we measure it with a ruler (or a tape measure). Assume that the bee is flying around in the air, and so this average position cannot be measured with uncertainty, and let us say that the total error bar of the bee's height measured by us with a ruler is about 1 mm. Let us say that the bee's position is

$x = 2001$ mm measured from the floor of the room. This means $x = 2001 \pm 0.5$ mm, so that total error bar is 1 mm. Now suppose that the bee was very close to the ceiling, and the height of the room is very accurately known: 2003.000 mm. What is the position of the bee relative to the ceiling? The answer is $-2003.000 + 2001 = -2$ mm. It is most definitely not -2.000 mm, for which the error bar would be ± 0.0005 mm. Such a small error bar would not make sense at all! How can the position of the bee become more precise just because the position is referenced to the ceiling instead of the floor? So, the answer should be just -2 mm. In general, if two numbers are added or subtracted, the error bar of the result is given by the larger of the two error bars associated with the two numbers added or subtracted ("Larger error bar wins"). Note that in addition or subtraction the number of significant figures of the result has nothing to do with the number of significant figures of the operands, as the above example shows.

Fourth, in adding or subtracting, can the answer be zero, even if two numbers added or subtracted are not exactly equal in magnitude? Yes! In science, "zero within the error bar" is a very important concept, and is encountered frequently in important measurements. An example: $3.0 - 2.99$. It appears that the answer should be $0.01 = 1 \times 10^{-2}$. This is meaningless, however. Why? Remember that the total error bar of 3.0 is 0.1, and so the result of the subtraction should have at least this much error bar. What is 0.01 compared to the error bar (uncertainty) 0.1? Nothing. So, the answer should be written as 0.0 instead of 0.01, i.e. this is the case of "zero within the error bar."

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