Quiz 4 Your Name: ____________________

20 minutes; G.-H. Sam Gweon, Phys 139A, UC Santa Cruz, Spring 2008

1. [15 points] Consider two operators $\hat{O}_1$ and $\hat{O}_2$, defined by

$$\begin{aligned} \hat{O}_1\psi(x) &= C \\ \hat{O}_2\psi(x) &= C\psi(x) \end{aligned}$$

Here C is a non-zero number.

(a) Is $\hat{O}_1$ a linear operator? How about $\hat{O}_2$?

(b) Explain your answers in (a). The best way is to provide a counter-example if it is not linear, or to prove the linear property if it is linear. If you already explained your answers in (a), move on to (c). A counter example here means a *particular* [and simple] example function $c_1\psi_1(x) + c_2\psi_2(x)$ that does not transform according to the rule of the linear operator: i.e. $\hat{O}(c_1\psi_1(x) + c_2\psi_2(x)) \neq c_1\hat{O}\psi_1 + c_2\hat{O}\psi_2$.

(c) Consider a non-relativistic quantum particle moving under the influence of a constant potential $V(\hat{x}) = C$. Which of the two operators above, $\hat{O}_1$ and $\hat{O}_2$, corresponds to $V(\hat{x})$ that enters the Schrödinger equation?

2. [5 points; This is an optional bonus problem. It is also a bit of a practice exam for the midterm. If you finish this problem in the given time, great. If not, you are welcome to submit a separate solution for this part by tomorrow afternoon.] As in the textbook, consider the square well potential

$$\begin{aligned} V(x) &= 0,\ 0 \leq x \leq a \\ &= \infty,\ otherwise \end{aligned}$$

We write the solution of the TISE (time-indep. Schrödinger eq.) for this problem as $\psi_n(x)$, $n = 1, 2, 3, ...$, and the corresponding energies E_n. For this problem, you do not need to specify what E_n actually is. However, you do need to remember [or derive] the functional *form* of $\psi_n(x)$ for (a), and at least the parity property of $\psi_n(x)$ for the rest of the problem.
IMPORTANT: Throughout this problem, treat constants A, B, C, D (see below) and energies E_n (see above) simply as symbols, *without* investigating how you can express them in terms of $a, \pi, \hbar$ and so on.

(a) Consider a normalized wave function at $t = 0$, $\Psi(x, 0) = A\sin^3(\frac{\pi}{a}x)$. Show that $\Psi(x, 0)$ is a linear combination of $\psi_1(x)$ and $\psi_3(x)$.
Here, all you need to show is that $\Psi(x, 0) = B\psi_1(x) + C\psi_3(x)$ where B, C are some numbers. The following rigonometric identities may be used: $\sin^2 x = \frac{1-\cos 2x}{2}$, $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$.

(b) What is the parity of $\Psi(x, 0)$ with respect to the center of the well $x = a/2$? By parity, I mean the even parity $\Psi \to \Psi$ or the odd parity $\Psi \to -\Psi$, when x is reflected w.r.t. $a/2$.

(c) Is there a definite parity of $\Psi(x, t)$? If so, what is it?

(d) What is $\langle\hat{x} + iD\hat{p}\rangle$ for $\Psi(x, 0)$? Here, D is a constant of dimension $[x/p]$. Does the same answer apply to $\Psi(x, t)$ as well?