Quiz 3 Your Name: ______________________

20 minutes; Δx and Δp; Phys 139A, UC Santa Cruz, Spring 2008

1. Consider a wave function

$$\psi(x) = A\left[\psi_g(x) + \psi_g(x-L)\right]$$

where $\psi_g(x) = \frac{1}{(2\pi\sigma^2)^{1/4}}\exp(-\frac{x^2}{4\sigma^2})$ is a *normalized* gaussian wave function with the following well-known properties: $\langle\hat{x}\rangle = 0$, $\Delta x \equiv \sigma_x = \sigma$, $\langle\hat{p}\rangle = 0$, and $\Delta p \equiv \sigma_p = \frac{\hbar}{2\sigma}$. We consider the case $L \gg \sigma$, which means that the wave function overlap between $\psi_g(x)$ and $\psi_g(x-L)$ is exponentially small. Therefore, we can make an *extremely accurate* approximation (let us call this a "no interference approximation" or "NIA" in short)

$$\psi_g(x)\hat{O}\psi_g(x-L) \text{ and } \psi_g(x-L)\hat{O}\psi_g(x) \approx 0$$

where $\hat{O} = \hat{x}^n$ or $\hat{p}^n$, and n is a small integer (say 0,1,2 just to be concrete). Using this approximation, answer the following questions for $\psi(x)$, to leading order in σ/L.

(a) [5 points] Show that the NIA means that $\langle\hat{O}\rangle \approx |A|^2\left[\langle\hat{O}\rangle_{g,0} + \langle\hat{O}\rangle_{g,L}\right]$ where $\langle\hat{O}\rangle \equiv \int_{-\infty}^{\infty} dx\psi(x)^*\hat{O}\psi(x)$, $\langle\hat{O}\rangle_{g,0} \equiv \int_{-\infty}^{\infty} dx\psi_g(x)^*\hat{O}\psi_g(x)$, and $\langle\hat{O}\rangle_{g,L} \equiv \int_{-\infty}^{\infty} dx\psi_g(x-L)^*\hat{O}\psi_g(x-L)$.

(b) [5 points] Assume that A is a positive real number. What is its value?

(c) [7 points] Find $\langle\hat{x}\rangle$, Δx, $\langle\hat{p}\rangle$, Δp, and $\Delta x\Delta p$.

(d) [3 points] Answer questions (b,c) for $\psi(x) = A\left[\psi_g(x+L) + \psi_g(x) + \psi_g(x-L)\right]$ with the same condition $L \gg \sigma$ and with the same type of NIA: $f(x)\hat{O}g(x) \approx 0$ where $f(x), g(x)$ are any two of $\psi_g(x+L), \psi_g(x), \psi_g(x-L)$ and $\hat{O}$ is as defined above.