Quiz 2

Phys 139A, Spring 2008, UC Santa Cruz

15 minutes; Probability flux

1. [4 points] The Schrödinger equation is $i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$ where $V(x)$ is a real potential, as usual.
   (a) Show the derivation of the following continuity equation from the above equation $\frac{\partial\rho}{\partial t} + \frac{\partial j}{\partial x} = 0$ where $\rho(x,t) = \Psi(x,t)^*\Psi(x,t)$ and $j(x,t) = \frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)$.
   (b) What is the physical dimension of j? [in terms of L (length), M (mass) and T (time) or in your familiar unit.]

2. [3 points] In three spatial dimensions, the continuity equation becomes $\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$ where $\vec{j}$ is the same as above except $\frac{\partial}{\partial x} \to \vec{\nabla}$. What is the physical dimension of $\vec{j}$? [Hint: In three dimensions, $\rho d\vec{x} = \rho dxdydz$ is probability.]

3. [3 points] Which of the following quantity has the same dimensions as $\vec{j}$ [note that j in 1 is really a vector, also] for both case 1 and case 2? Explain your answer briefly. [Hint: the best way to answer this question is to look only at the continuity equations]
   (A) $\frac{dP}{dt}$ (B) $\rho\vec{v}$ (C) $\vec{\nabla}\rho$
   where $P = \int d\vec{x}\rho(\vec{x},t)$ is the probability and $\vec{v}$ is some velocity.

Your Name:

Your Answers: