Quiz 1 + Math/Physics Questionnaire

25 mins; closed-everything [not even looking up tables, for this one]

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1. [Quiz – 5 points] Consider a wave of light with angular frequency ω impinging on your eye, depositing total energy E. What does it mean when a physicist says that this light consists of N photon particles? Your answer should indicate (1) what the energy of a single photon is and (2) what the total energy E is in terms of that and N.

Below is a set of math/physics questions **for information gathering, not for evaluation**. Quantum Mechanics, *the* central course in undergrad physics curriculum [in my opinion], (1) is quite mathematical and (2) builds on concepts learned in classical physics. I designed the following questions so that I get a sense of your level of preparedness. Analyzing your answers on these questions will greatly ease my teaching, and thus, most importantly, your learning. So, please provide your answers to the best of your knowledge, even if some questions may look "trivial" [never true, really].

1. Given $\sin(x) = 2/3$, evaluate $\cos(2x)$.
2. Evaluate the following integrals.
   (a) $\int_{-\infty}^{\infty} dx x \cos(x) \exp(-x^2)$ [You should give the answer <u>without</u> doing any calculations for this one.]
   (b) $\int^{x} dy y \cos(y)$ [Indefinite integral; Show clearly the step of the integration by parts.]
3. Write down (a) the first four non-zero terms of the Taylor expansion for $\exp(x)$, and (b) the first two terms of the Taylor expansion for $\cos(x)$.
4. $\vec{x} = \begin{bmatrix} 1 \\ 2i-1 \\ 3 \end{bmatrix}$. Write down the expressions for $\vec{x}^T$ and $\vec{x}^\dagger \equiv \vec{x}^{T*}$.
5. What are the eigenvalues for the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$?
6. Write down the form of the general solution for $f(x)$ in the differential equation $\frac{d^2 f(x)}{dx^2} = -\kappa^2 f(x)$, where κ is a number.
7. The normalized Gaussian distribution function centered at 0 is $\frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{x^2}{2\sigma^2})$. What function [or "generalized function"] does this become when $\sigma \to 0$?

   [For the physics questions below, simply state answers from your memory, as you probably do not have enough time to derive all formulae. The last one is an advanced question. Don't worry if you never heard about the virial theorem.]

8. [E&M] There is a current loop of radius R, with current I. What is the magnetic moment $\vec{\mu}$ of this loop? Indicate both the magnitude and the direction. What is the potential energy V when this current loop is placed in an external $\vec{B}$ field?
9. [Classical Mechanics] A particle of mass m is making a circular motion at distance r from origin, with angular velocity $\omega = \frac{2\pi}{T}$. Express the acceleration a, the angular momentum $L = |\vec{L}|$, and the kinetic energy $T = \frac{1}{2}mv^2$ in terms of m, r and ω.
10. [Classical Mechanics] In classical mechanics, the time-averaged potential energy $\langle V \rangle$ and the time-averaged kinetic energy $\langle T \rangle$ have a simple relationship to each other when $V(\vec{r}) = Ar^n$, wher A is a constant, $r = |\vec{r}|$, and n is an integer. This goes by the name "virial theorem." [We will discuss the quantum analogue later.] For a simple harmonic oscillator problem for which $n = 2$, what is the relationship between $\langle V \rangle$ and $\langle T \rangle$? When $n = -1$ (Coulomb's law of electrostatics or Newton's law of gravitation), what is the relationship?

Your Name:

Answer to Quiz:

Answers to M/P Questionnaire: