§. Observables and $[\hat{x}, \hat{p}] = i\hbar$

Q

$\hat{x}$ is an observable

$\hat{p}$ is an observable

$[\hat{x}, \hat{p}]$ is an observable

why is it ~~complex~~ (purely imaginary)?

A

Actually $[\hat{x}, \hat{p}]$ is not an observable.
As we will see an observable has to be a hermitian operator. For example

$\hat{x}^\dagger = \hat{x}$, $\hat{p}^\dagger = \hat{p}$

However notice that

$$(\hat{x}\hat{p})^\dagger = \hat{p}^\dagger \hat{x}^\dagger$$
$$= \hat{p}\hat{x} \neq \hat{x}\hat{p}$$

So, $\hat{x}\hat{p} + \hat{p}\hat{x}$ is an observable

$$(\hat{x}\hat{p} + \hat{p}\hat{x})^\dagger = \hat{x}\hat{p} + \hat{p}\hat{x}$$

but $\hat{x}\hat{p} - \hat{p}\hat{x}$ is not an observable.

$$(\hat{x}\hat{p} - \hat{p}\hat{x})^\dagger = -(\hat{x}\hat{p} - \hat{p}\hat{x})$$