Homework 5

Phys 139A, Spring 2008, UCSC

Due May 13, 10 am

1. [20 points; Gaussian Wave Packet] This problem is a re-visit of the solution of Prob. 2.22 provided in the lecture note. By far, the trickest part of that solution is the part (d) to evaluate $\langle \hat{p}^2 \rangle$. In particular, it might seem strange that $\langle \hat{p}^2 \rangle$ has no time-dependence. Let us see how we can understand this more easily from a much more general perspective.

   (a) Show that for *any* wave function $\Psi(x,t)$ for a free particle, $\langle \hat{p}^2 \rangle = \int_{-\infty}^{\infty} dk |\phi(k)|^2 \hbar^2 k^2$ at *any* time, using the general solution $\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \phi(k) \exp(i(kx - \omega(k)t))$ where $\omega(k) \equiv \hbar k^2/(2m)$. [Hint: $\int_{-\infty}^{\infty} dx \exp(i(k-k')x) = 2\pi\delta(k-k')$ will be useful.]

   (b) Comment on the reasons why there is no time dependence in $\langle \hat{p}^2 \rangle$. If $\hat{p}^2$ is replaced by any arbitrary function $f(\hat{p})$, would there be still no time dependence in $\langle f(\hat{p}) \rangle$? [For the first part, there are two reasons! For the second part, you may assume that $f(\hat{p})$ is analytic and so it can be written as $f(\hat{p}) = \sum_{n=0}^{\infty} a_n \hat{p}^n$. You should prove to yourself that $f(\hat{p}) \exp(ikx) = f(\hbar k) \exp(ikx)$.]

   (c) Go back to Prob. 2.22. Using the above result in (a), prove that $\langle \hat{p}^2 \rangle = \hbar^2 a$ at *any* time, using either $\phi(k)$ or $\Psi(x,0)$ given in the solution.

2. [10 points] Problem 2.21

3. [20 points] Problem 2.47