Homework 2

Phys 139A, Spring 2008, UCSC

Due April 18, 1 pm

In 4,5 below, $\Delta x \equiv \sigma_x$ and $\Delta p \equiv \sigma_p$.

1. [10 points; Unstable particle described by a complex potential (this case should be regarded as a very rare exception to the standard case, where the potential is real); Time-Energy uncertainty relation] Problem 1.15

2. [10 points] Problem 1.16

3. [20 points] Problem 1.17

4. [10 points; Uncertainty relation and a macroscopic object] Suppose you have a tennis ball, which is about 60 gram in mass and 6 cm in diameter. Let us say that this ball is moving at a slow speed of 1 mm / sec. Suppose that we can specify the position of the ball, namely the location of the center of mass, very accurately [to an atomic precision], $\Delta x = 0.1$ nm. Assuming the minimum uncertainty relation, what is the uncertainty of momentum Δp? Show that both $\Delta x/x$ (here x is the scale of the object and can be taken as the diameter of the ball) and $\Delta p/p$ are very negligible, so that for all practical purposes Δx and Δp can be regarded as zero.

5. [20 points; Messy uncertainty wave function] The uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$ means that $\Delta x \Delta p = \frac{\hbar}{2}$ is possible, as is the case for a [nice] "minimum uncertainty wave function" as in prob. 1.9, or that there is a possibility that the wave function can be messed up so that $\Delta x \Delta p \gg \frac{\hbar}{2}$ (let us call this type of wave function a "messy uncertainty wave function"; we will see some more examples later, for instance, in prob. 2.22). In this problem, we will build a simple example wave function for which $\Delta x \Delta p \gg \frac{\hbar}{2}$. Consider the following wave function at $t = 0$

$$\Psi(x,0) = A\,[g(x) + g(x-L)]$$

where $g(x) = \frac{1}{\sqrt[4]{2\sigma^2\pi}} \exp(-\frac{x^2}{4\sigma^2})$. Note that the wave function $g(x)$ represents a normalized Gaussian wave function with standard deviation σ. We consider the case $L \gg \sigma$, say $L/\sigma = 100$ to be more concrete.

   (a) Consider the integral $\int_{-\infty}^{\infty} dx x^n g(x) g(x-L)$ where n is an integer (0, 1, or 2). Show that for all practical purposes, this integral is totally negligible compared to a number of $O(\sigma^n)$ [Hint: notice that the integral is proportional to $\exp(-\frac{L^2}{8\sigma^2})$].

   (b) Ignoring the type of integral in (a), determine the normalization constant A (assume it to be real), $\langle \hat{x} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p} \rangle$, $\langle \hat{p}^2 \rangle$, Δx and Δp. Verify that $\Delta x \Delta p \gg \frac{\hbar}{2}$. For this part of the problem, the recommended, and the simplest, way to get your answers is not to explicitly evaluate integrals but simply to use known results for a gaussian wave function as we studied in prob. 1.9. That is, for this part of the problem, you should not need to write out the functional forms of $g(x)$ and $g(x-L)$.