Homework 1

Phys 139A, Spring 2008, UCSC

Due April 11, 5 pm

1. [10 points] Problem 1.11
2. [10 points] Problem 1.14
3. [10 points; The zero point energy of a harmonic oscillator from the Uncertainty Principle and the virial theorem.]
   Consider the Hamiltonian operator for a harmonic oscillator

$$\hat{H} = \hat{T} + \hat{V} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

   Assume that, for a wave function (unspecified for the purpose of this problem), the following properties hold.

$$\begin{aligned} \langle\hat{x}\rangle &= 0 \text{ (symmetric probability density)} \\ \langle\hat{p}\rangle &= 0 \text{ (the same)} \\ \langle\hat{T}\rangle &= \langle\hat{V}\rangle \text{ (QM version of the Virial theorem)} \\ \sigma_x\sigma_p &= \hbar/2 \text{ (the minimum uncertainty relation)} \end{aligned}$$

   Using these properties, and the definitions $\sigma_x = \sqrt{\langle\hat{x}^2\rangle - \langle\hat{x}\rangle^2}$ and $\sigma_p = \sqrt{\langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2}$, show that $\langle\hat{H}\rangle = \frac{1}{2}\hbar\omega$. This is the ground state energy of a harmonic oscillator.