



Lecture 19

Magnetic Order

Love thy neighbors

Different Kinds of Magnetic Order

- **Ferro-magnetism** (Fe, CrO_2 , EuO , etc.)
- **Anti-ferromagnetism** (MnO , FeF_2 , HTSC cuprates, etc.)
- **Ferri-magnetism** (Fe_3O_4) ...
- Note
 1. Magnetic orders happen at fairly high temperatures (100 K ~ 1000 K) – i.e. internal field is very high due to Coulomb interaction
 2. The above are $B=0$ properties, while the following designate response to finite B :
 - ❑ Para-magnetism (Curie, Van Vleck, Pauli)
 - ❑ Dia-magnetism (Larmor, Landau, London)

Interaction between Magnetic Ions

- Dipole-dipole interaction is very small

$$\vec{B} = \frac{3(\vec{m} \cdot \vec{r})\vec{r} - \vec{m}}{r^3} \quad (\text{cgs})$$

$$\mu_B \approx 6 \times 10^{-9} \text{ eV/gauss} \approx 9 \times 10^{-21} \text{ erg/gauss}$$

$$B \sim \frac{\mu_B}{r^3} \sim \frac{9 \times 10^{-21}}{(3 \text{ \AA})^3} \sim \frac{10^{-21}}{3 \times 10^{-24}} \sim 1000 \text{ gauss}$$

$$E \sim \mu_B B \sim 6 \times 10^{-6} \text{ eV} \sim 0.006 \text{ meV} \sim 0(0.01) \text{ K}$$

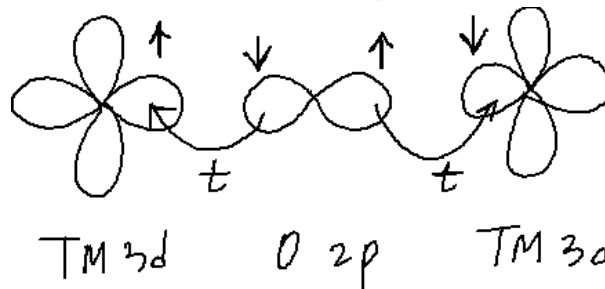
- Magnetic exchange energy results from Coulomb interactions: hopping (t) and exchange interactions
- Two most important Hamiltonians:

Heisenberg $H = -\sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$

Hubbard $H = \text{Kinetic Energy} + \text{Exchange Coulomb Energy}$

Sign of J in Heisenberg Model

- Positive like “Hund’s rule” ($J > 0$): Equal spins repel less
- **Super-Exchange** ($J < 0$): TM ions (such as ions of Fe, Mn, Cu etc) interact via anions



- Double exchange ($J > 0$), RKKY (J can be random), ...

Ferromagnetism due to local spins

- Consider Heisenberg Hamiltonian with only nearest-neighbor interaction (now $\langle i,j \rangle$ means only nearest neighbor pairs, not all possible pairs) and $J > 0$

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

- This Hamiltonian is very difficult to solve. One can however show that all spins pointing in the same direction is the lowest energy state. For instance for spin $1/2$, it is easy to show

$$H |\psi\rangle = -NJ/4 |\psi\rangle \quad |\psi\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\uparrow\uparrow\rangle$$

where z direction is any random direction. (Hint:

$$\text{use } S_x = \frac{1}{2} (S_+ + S_-) \quad S_y = -\frac{i}{2} S_+ + \frac{i}{2} S_-)$$

Ferromagnetism due to local spins

- Clearly the ground state is one in which all spins are aligned (symmetry breaking) and the excited state is in one in which spins are flipped (magnon or spin wave)
- At finite T , each spin will “fluctuate” around its mean value (even at $T=0$ due to quantum fluctuations)
- An elementary approach is then to ignore these fluctuations and apply a “mean field” theory

Mean field theory of FM (Weiss)

- There is an enormous internal field due to magnetization ($\lambda \gg 1$):

$$\lambda \mu_0 \vec{M} \quad \vec{B} = \lambda \mu_0 \vec{M} + \vec{B}_{loc}$$

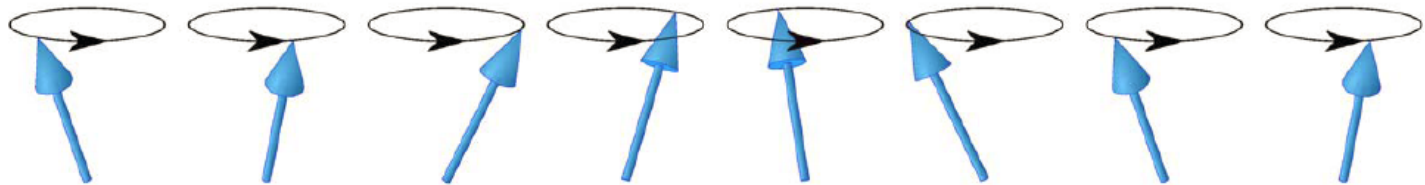
- At high T, Curie-Weiss Law: $\chi = \frac{C}{T - T_c}$
- At $T < T_c$, spontaneous magnetization
- $T = 0$: Saturation without any external field $M = N\mu$
- Provides basic picture for the FM transition but ...
 - T-dependence near T_c (wrong):

$$M \propto \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}} \quad \chi \propto (T - T_c)^{-1}$$

- T-dependence near $T = 0$ (wrong):

$$M = N\mu \left(1 - 2 \exp\left(-\frac{2T_c}{T}\right)\right)$$

Spin Wave and Bloch $T^{3/2}$ law in FM



<http://upload.wikimedia.org/wikipedia/commons/0/01/FerromagneticMagnon.svg>

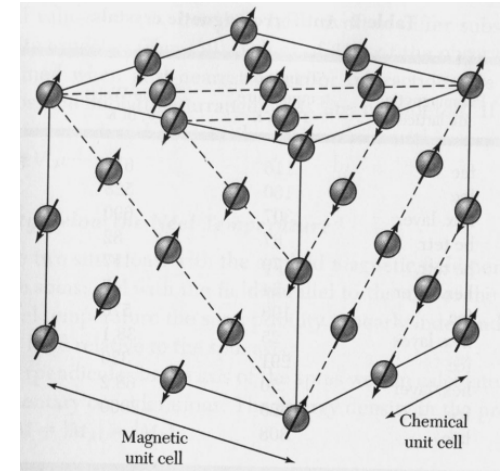
- Quantum Mechanically, spin wave means excited states with one spin flipped
- Easiest way to consider this is to do Classical Mechanics (like phonon problem)
- We know that $k=0$ solution must exist with energy = 0 (like the acoustic phonon case)
- FM spin wave has the property: $\omega \propto k^2$
- This leads to Bloch's law near $T=0$:

$$C \propto T^{3/2} \quad \vec{M} \text{ reduction} \sim T^{3/2}$$

Antiferromagnetism

- Spins align anti-parallel to each other ($J < 0$ due mostly to the Anderson super-exchange)
- Different from FM in that
 - AFM state is not an eigenstate!
 - But thermodynamically favored state at $T < T_N$
 - Magnetic unit cell differs from chemical unit cell
 - Spin wave is really like phonon:

$$\omega \approx \omega_{ex} |ka| \quad k \rightarrow 0$$



Mn²⁺ ions in MnO (from Kittel)

High T susceptibility of FM or AFM due to local spins

$$\chi \sim 1/(T - \Theta)$$

(FM: $\Theta \sim O(T_C)$, Curie Temperature)

$$\chi \sim 1/(T + \Theta)$$

(AFM: $\Theta \sim O(T_N)$, Néel Temperature)

Θ is not exactly T_C (or T_N) because the theory presented here is very simple (only nearest neighbor interaction and mean-field)

Itinerant Ferromagnetism

- One of the mechanism for metallic FM (as in Fe)
- Due to exchange Coulomb interaction
- Exchange interaction favors spin polarization but kinetic energy favors no spin polarization (**competition**)
- The onset of the FM is given by Stoner criterion in perturbation MF theory

$$\frac{g(E_F)}{N} \cdot U = 1$$

Stoner Ferromagnetism

- Just like Pauli Para-magnetism theory, but with internal field

$$E_{\vec{k}\uparrow} = E_{\vec{k}} + U n_{\downarrow} \quad \text{Exchange energy } U > 0$$

$$E_{\vec{k}\downarrow} = E_{\vec{k}} + U n_{\uparrow}$$

$$M = \mu_B (n_{\uparrow} - n_{\downarrow})$$

$$E_{\vec{k}\uparrow} = E_{\vec{k}} + U \cdot \frac{n}{2} - U \frac{M}{2\mu_B}$$

$$n = n_{\uparrow} + n_{\downarrow}$$

$n, n_{\uparrow}, n_{\downarrow}$: # densities

$$E_{\vec{k}\downarrow} = E_{\vec{k}} + U \cdot \frac{n}{2} + U \frac{M}{2\mu_B}$$

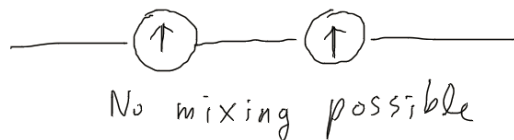
- Just like the last quiz, except self-consistency needs to be imposed (source M is the same as the result, as in Weiss MF theory) – left for reader

$$\frac{g(E_F)}{N} \cdot U = 1$$

Hubbard Model and AFM

- Hubbard Model provides a basis for the Stoner FM (small U)
- Hubbard Model also provides a basis for an AFM in the opposite limit of large U

Hubbard model has an AFM when
 $U \rightarrow \infty$



E lowered by $-\frac{t^2}{U}$

Also, Mott-Hubbard Insulator
(cf. Midterm I (e))

i.e., AFM MH Insulator