Lecture 18
Magnetism

Magnetism is probably the most complicated phenomenon in condensed matter physics.

Main driving forces are Spin and Coulomb interaction.

Origin of Magnetic Moment in Solids

- Electrons are the main source

  (Nuclear spins do come in handy as in NMR)

- Magnetic moment
  - $\mu = -\mu_B (\mathbf{L} + g_0 \mathbf{S})$, $g_0 = 2.0023\ldots \approx 2$
  - $\mu_B = e\hbar/(2m)$
    - μ_B = 9.27e-24 J/T (SI) or 5.8e-9 eV/gauss
    - For nucleons, reduction by ~ 2000
    - Remember, B field ~ 100 T = 1e6 gauss is the strongest field that one can generate, so magnetic perturbation energies are quite small.

Some Atomic Physics …

- **Spin-orbit coupling**:
  - $H_{LS} = \mathbf{L} \cdot \mathbf{S}\, e^2/(r^3 8\pi\varepsilon_0 m^2 c^2)$
  - Goes like ~ α^2 x unperturbed energy
  - The heavier the atom, the greater $\langle H_{LS} \rangle$
  - The closer the orbital is to the atom, the greater $\langle H_{LS} \rangle$
  - Quite important for the band structure of semiconductors (44 meV for Si, 0.34 eV for GaAs)
- **Zeeman interaction**:
  - $H_B = -\mu \cdot \mathbf{B} = -\mu_B (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}$
- For very light atoms (Li, e.g.), the Zeeman term can be more important than the spin-orbit coupling.
- **For most solid problems, the spin-orbit coupling is more important than the Zeeman term.**
  - Treat spin-orbit first, and then, treat Zeeman as a perturbation.
  - Good Quantum Numbers: J, L, S, J_z
  - $\langle H_B \rangle = g(L,S,J)\, \mu_B J_z B$ (**g: Lande g-factor**; eq. 7.10; see Sakurai)

$\mu_{eff} = g\, \mu_B J_z$

Ref: QM books (Griffiths, Sakurai …)

- Next week final exams
  - Note that report due by final lecture
  - Tuesday group and Thursday group
  - Presentation computer?
  - Video tape operator?
- Final lecture, 3/19 9 am
  - Final homework (9) due
  - Final report due

Hund's Rule for Atomic Spectra

- Empirical rule that works pretty well for 3d Transition metal (TM) and 4f rare earth (RE) ions
- Many electrons in an un-filled orbital
  - First rule: maximize S
  - Second rule: maximize L
  - Third rule: J = |L-S| if less than half full and J = L+S if more than half full
  - Notation: $^{2s+1}L_J$
- E.g., V^{3+} : $3d^2$ $^3F_2$, Fe^{2+} : $3d^6$ $^5D_4$, Mn^{2+} : $3d^5$ $^6S_{5/2}$
- Roughly, the physics is due to Coulomb interaction and spin-orbit interaction

Ref: Slater "Quntum Theory of Atomic Structure," Landau and Lifshitz

Curie Paramagnetism

- Localized magnetic moments in response to external B field
- $M = \chi H$
- Curie law at high T, small B: **$\chi = C/T$**
  - $C = N p^2 \mu^2_B \mu_0 / (3 k_B)$ (N is volume density)
  - $p = g [J(J+1)]^{1/2}$
- Saturation at low T, large B: $M = Ng\mu_B J$
- p agrees well for RE, but not so for TM
- For TM ions, $p = 2 [S(S+1)]^{1/2}$ works much better!?

H&H, 7.2.3

Quenching of Orbital Moment in Transition Metal Ions in Crystal

- Empirically observed (table 7.2)
- Effect of Crystal Field (large ~ 1 eV in TM ions)
- L_z is not a good quantum number
- $< L_z >$ (also for x,y) is often zero

Pauli Para-magnetism of Metals

- Response of Fermi sea to the magnetic field
- Spin-up electron's energy goes up and spin-down … down, both by $\mu_B B$
- $M = \mu_B(n_\uparrow - n_\downarrow) = \mu_B^2 g(E_F) B$ (g is per volume, but for both spins)
- $\chi = M / H \approx \mu_0 \mu_B^2 g(E_F)$ (T independent)
- $\chi \sim \mu_0 \mu_B^2 N/E_F$ (N is volume density), i.e. weakened by T/T_F (quantum behavior – only those at E_F respond) compared to Curie susceptibility (classical behavior – all spins respond)

Diamagnetism

- Lenz's law
- Landau Diamagnetism
  - Orbital Motion of Electrons
  - For free electron, $\chi_{Landau} = -\chi_{Pauli}/3$
  - For effective mass m*, $\chi_{Landau} / \chi_{Pauli} \sim (m/m^*)^2$

    i.e. $\chi_{Landau} >> \chi_{Pauli}$ (semi-cond) and $\chi_{Landau} << \chi_{Pauli}$ (renormalized metal)
- Larmor (Langevin) Diamagnetism due to closed shell ions ($\mathbf{A} = -\mathbf{r}\times\mathbf{B}/2$, $H = (\mathbf{p}+e\mathbf{A})^2/2m$, The last term of $H \sim r^2B^2$)
- For non-magnetic materials, often it is necessary to consider <u>all</u> susceptibilities (Pauli, Landau, Larmor, and Curie – often due to magnetic impurities)

H&H, 7.3