Lecture 15,16
Semiconductors – continued

Effective mass

1. At band maximum or minimum, the energy band can be written as $\propto k^2/m^*$, where m^* is the "effective mass," which can be defined as $\hbar^2[d^2\varepsilon(\mathbf{k})/dk^2]^{-1}$.
2. The definition above is useful mainly for semiconductors and semi-metals.
3. m^* tends to be small ~ a tenth of the mass of free electron (i.e. actual excitation is lighter than free electron). [HW]
4. For metals, energy dispersion near E_F is approximated as linear – effective mass m^* is then defined in terms of v_F ($v_F = v_{F0}/m^*$) and it tends to be larger than the mass of free electron (i.e. actual excitation is heavier than free electron), due to the electron-electron interaction or a "tight-binding-like" nature of band, both important for TM or RE materials.

Note

Charge Carriers in a Semiconductor

- Electrons in conduction band

- Holes in valence band

- Intrinsic Semiconductors: Carriers are created by excitation across the energy gap E_G

- Extrinsic Semiconductors: Carriers are provided by impurities (donors, acceptors)

Donors and Acceptors

- Consider Si or Ge, both with 4 valence electrons
- Substitutional impurities with 5 valence electrons (P, As) are donors
- Substitutional impurities with 3 valence electrons (B, Al) are acceptors
- Donors (acceptors) create impurity states just below (above) the conduction (valence) band, at energy ~ O(10) meV

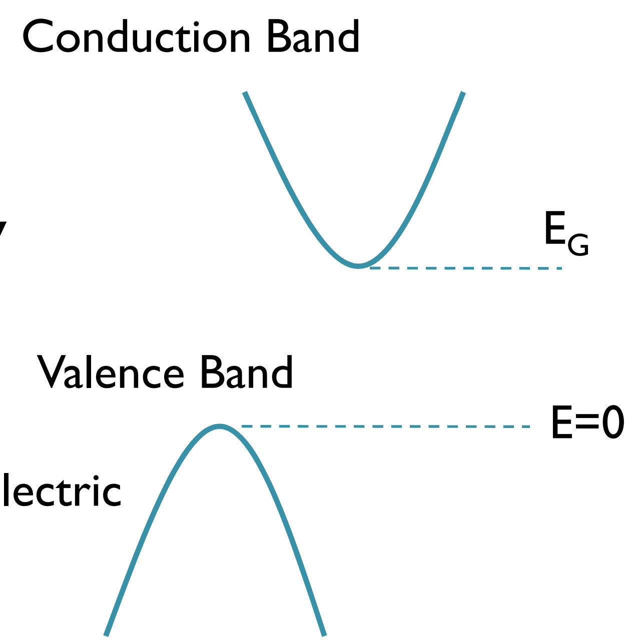
Note

Questions

- For a semiconductor, assume that μ is well within the gap (compared to k_BT). I.e., $\beta\ (E_G - \mu)$, $\beta\ \mu >> 1$. Show, from the free electron theory, that
- Electron density: $\mathbf{n = N_c \exp(\beta\ (\mu - E_G))}$

  $N_c = 2\ (m^*_e k_B T / 2\pi\hbar^2)^{3/2}$
- Hole density: $\mathbf{p = N_v \exp(-\beta\ \mu)}$

  $N_v = 2\ (m^*_h k_B T / 2\pi\hbar^2)^{3/2}$
- Consider an electron contributed by the donor ion. Considering a Bohr theory of a hydrogen problem (but with mass $m^*_e \sim 0.1\ m_e$ and dielectric constant $\varepsilon \sim 10$), show that the binding energy is about a thousandth of the hydrogen problem, i.e. ~ 10 meV. Mark this energy in the above diagram.



Note

Charge Carrier Population (General)

- Define energy 0 = top of valence band
- Electron density: $\mathbf{n = N_c \exp(\beta(\mu - E_G))}$

  $N_c = 2\,(m_e k_B T/2\pi\hbar^2)^{3/2}$
- Hole density: $\mathbf{p = N_v \exp(-\beta\mu)}$

  $N_v = 2\,(m_h k_B T/2\pi\hbar^2)^{3/2}$
- $np = N_c N_v \exp(-\beta E_G) = n_i p_i = n_i(T)^2 = p_i(T)^2$

  Law of mass action

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Note

Charge Carrier Population (Intrinsic Semiconductors)

- $n_i = p_i$
- $n_i = p_i = (N_c N_v)^{1/2} \exp(-\beta E_G/2)$
- <u>Intrinsic carrier density</u>:

  $n_i = p_i \sim 10^{16}$ (Si) and 10^{19} (Ge) m^{-3}

  (fewer than in metals by ~ 10 orders of magnitude or more)
- <u>Chemical Potential for Intrinsic Semi-cond</u>:

  $$\mu = \tfrac{1}{2} E_G + \tfrac{3}{4} k_B T \ln (m_h/m_e)$$

Warning: In semi-conductor literature, the chemical potential is very often referred to as the "fermi level." This is an unfortunate and un-recommended mis-nomer, as there is no Fermi surface!

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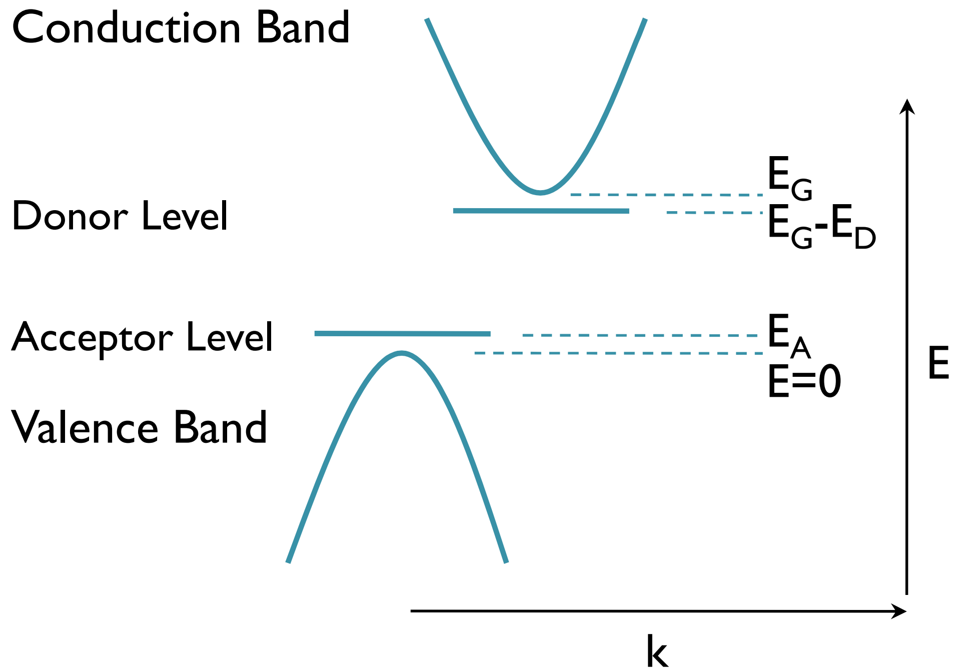
Note

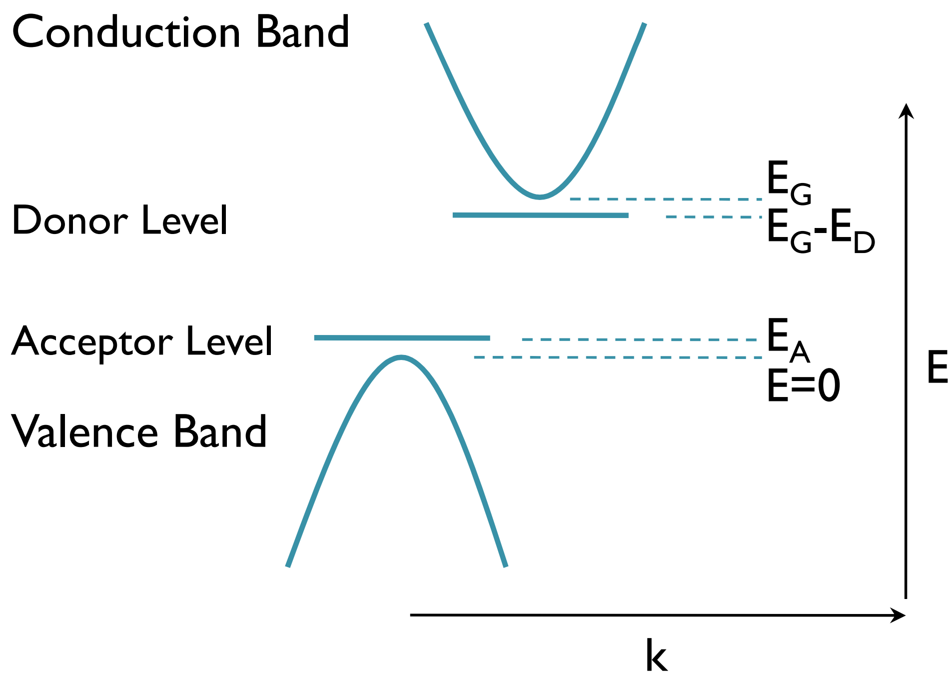
Charge Carrier Population (Extrinsic Semiconductors)



At T = 0

- Conduction Band A. is full B. is empty C. can have some electrons
- Valence Band is full
- Donors may have some electrons to spare, acceptors holes

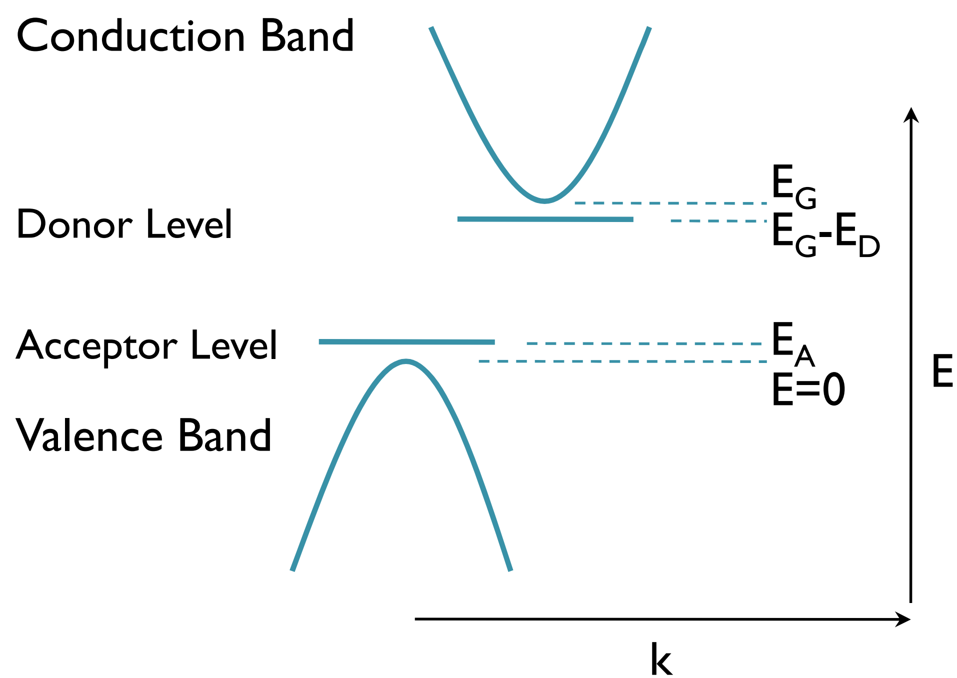
Charge Carrier Population (General)



At T = 0

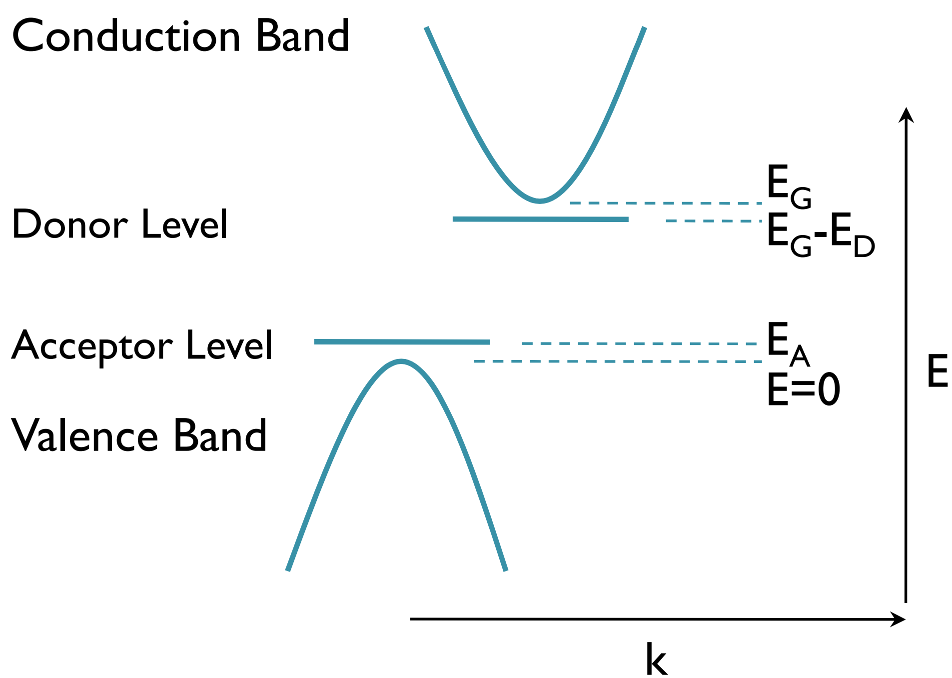
- Conduction Band is empty
- Valence Band is full
- Donors may have some electrons to spare, acceptors holes

Charge Carrier Population Where is Chemical Potential at T=0?



- Common n-type (More donors than acceptors; majority carriers are electrons from donors, and minority carriers are holes from acceptors): A. $E_G/2$ B. E_G-E_D C. E_A
- Common p-type: E_A
- Pure n-type: A. $E_G/2$ B. E_G-E_D C. $E_G-0.5E_D$
- Pure p-type: $E_A/2$
- Intrinsic: $E_G/2$

Charge Carrier Population
Where is Chemical Potential at T=0?



- Common n-type (More donors than acceptors; majority carriers are electrons from donors, and minority carriers are holes from acceptors): E_G-E_D
- Common p-type: E_A
- Pure n-type: E_G-0.5E_D
- Pure p-type: $E_A/2$
- Intrinsic: $E_G/2$

Extrinsic Semiconductors (n-type)

- For $T \approx 0$

  $\mu \approx E_G - E_D$ (common n-type: small # of acceptor impurities)

  $E_G - 0.5\ E_D$ (pure n-type: absolutely no acceptors)

  $n \approx N_c \exp(-\beta E_D)$ (common n-type)

  $N_c \exp(-\beta E_D/2)$ (pure n-type)

- $n >> n_i$: electron is **majority** carrier
- $p << p_i << n$ (recall $np = n_i p_i$): hole is **minority** carrier
- As T increases (~ E_D), all donors lose electrons

  $n \approx N_D - N_A$ (N_A: acceptor # density, N_D: donor # density)

  $\mu \approx E_G - k_B T \ln(N_c / (N_D - N_A)) \equiv \star$

- Intrinsic behaviors : at even higher T
- $\mu : E_G - [0.5]\ E_D \rightarrow \star \rightarrow 0.5\ E_G$ as $T \uparrow$
- $n(T) : \exp(-\beta\ [0.5]\ E_D) \rightarrow N_D - N_A \rightarrow \exp(-\beta\ 0.5\ E_G)$

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Extrinsic Semiconductors (p-type)

- For $T \approx 0$

  $\mu \approx E_A$ (common p-type: small # of donor impurities)

  $0.5\ E_A$ (pure p-type: absolutely no donors)

  $p \approx N_v \exp(-\beta E_A)$ (common p-type)

  $N_v \exp(-\beta E_A/2)$ (pure p-type)

- $p >> p_i$: hole is **majority** carrier
- $n << n_i << p$ ($np = n_i p_i$): electron is **minority** carrier
- As T increases ($\sim E_A$), all acceptors lose holes

  $p \approx N_A - N_D$

  $\mu \approx k_B T \ln (N_v / (N_A - N_D)) \equiv$ ☆

- Intrinsic behaviors : at even higher T
- μ : [0.5] $E_A \rightarrow$ ☆ $\rightarrow 0.5\ E_G$ as $T \uparrow$
- $n(T)$: $\exp(-\beta\ [0.5]\ E_A) \rightarrow N_A - N_D \rightarrow \exp(-\beta\ 0.5\ E_G)$

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Example of temperature dependence of an n-type semi-cond

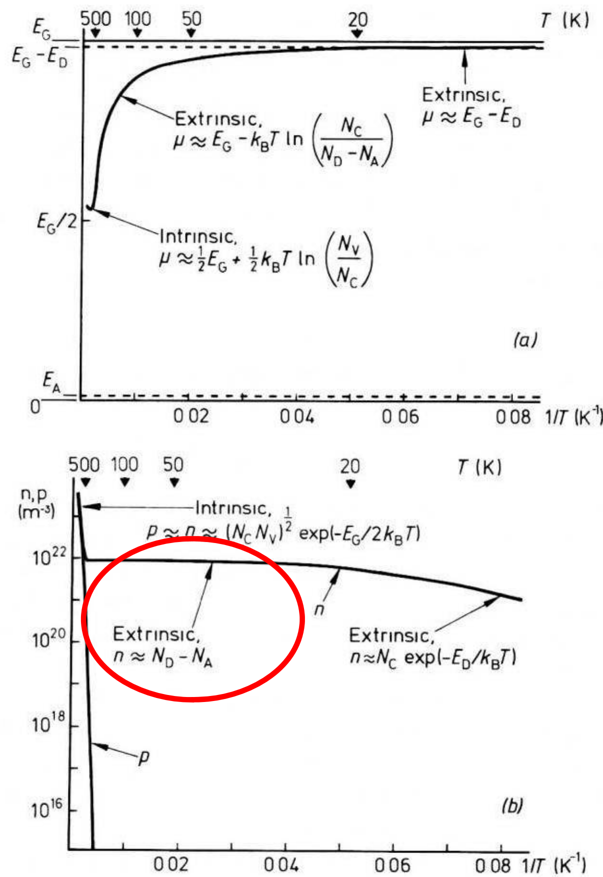

Fig. 5.6 Variations of (*a*) the Fermi level μ and (*b*) the electron and hole concentrations (note the logarithmic scale) with $1/T$ for an n-type semiconductor containing a significant number of acceptor impurities. The figure was calculated for a germanium semiconductor with $N_D = 10^{22}\ m^{-3}$, $E_D = 0.012$ eV, $N_A = 10^{21}\ m^{-3}$ and $E_A = 0.010$ eV; the scale at the top shows temperature values for this case

Transport Properties ("hole" is real)

- Equation of Motion: $m^* (d/dt + 1/\tau)\, \mathbf{v}_d = \mathbf{F}$ (as we used in metal)
- Hall Effect (for single type of carrier) in B field

  $R_H \equiv E_y/(B_z j_x) = -1/(ne)$ or $1/(pe)$ [sign !]
- Thermo-electric Effect can probe the sign of the carrier charge as well
- Cyclotron Resonance in B field: $\omega_c = eB/m^*$ (SI unit)
- Conductivity

  $\sigma = ne\mu_e + pe\mu_h$

  $\mu_e = e\tau_e/m^*_e = v_{d,e}/E$, and similarly for μ_e

  μ_e, μ_h: mobility

  1. useful concept for semiconductors
  2. characterizes "quality" rather than "quantity" (n or p)
  3. not important for T dependence of σ (determined by exponentials in n or p – see slide "Charge Carrier Population")

Note

Charge Carrier Motion (n-type)

- Definition

$$n = n_0 + n'(x,t), \quad p = p_0 + p'(x,t)$$

n_0: equilibrium, $n'(x,t)$: non-equilibrium
p_0: equilibrium, $p'(x,t)$: non-equilibrium

- Continuity Equation

$$\frac{\partial n}{\partial t} + \frac{\partial J_e}{\partial x} = g - r \qquad \frac{\partial p}{\partial t} + \frac{\partial J_h}{\partial x} = g - r$$

J: number current not charge current

g: generation rate (source term)

r: recombination rate (sink term)

- Majority Carrier Equation (for no ext. field)

$$\frac{\partial n'}{\partial t} = -\frac{n' - p'}{\tau_D} + \frac{\lambda_D^2}{\tau_D}\frac{\partial^2 n'}{\partial x^2}$$

Majority carriers screen non-equilibrium charge fast :
τ_D ~ psec (10^{-12} sec), λ_D ~ 100 Å

- Minority Carrier Equation

$$\frac{\partial p'}{\partial t} = -\frac{p'}{\tau_n} + D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h E \frac{\partial p'}{\partial x}$$

Minority carriers have slower response :
$\tau_n \sim 10^{-7}$ sec, $\lambda = \sqrt{(D_h \tau_n)} \sim 10\ \mu$m

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Charge Carrier Motion (p-type)

- Definition

$$n = n_0 + n'(x,t), \quad p = p_0 + p'(x,t)$$

n_0: equilibrium, $n'(x,t)$: non-equilibrium
p_0: equilibrium, $p'(x,t)$: non-equilibrium

- Continuity Equation

$$\frac{\partial n}{\partial t} + \frac{\partial J_e}{\partial x} = g - r \qquad \frac{\partial p}{\partial t} + \frac{\partial J_h}{\partial x} = g - r$$

J: number current not charge current

g: generation rate (source term)

r: recombination rate (sink term)

- Majority Carrier Equation (for no ext. field)

$$\frac{\partial p'}{\partial t} = -\frac{p' - n'}{\tau_D} + \frac{\lambda_D^2}{\tau_D}\frac{\partial^2 p'}{\partial x^2}$$

Majority carriers screen non-equilibrium charge fast :
τ_D ~ psec (10^{-12} sec), λ_D ~ 100 Å

- Minority Carrier Equation

$$\frac{\partial n'}{\partial t} = -\frac{n'}{\tau_p} + D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e E\frac{\partial n'}{\partial x}$$

Minority carriers have slower response :
τ_p ~ 10^{-7} sec, $\lambda = \sqrt{(D_e\tau_p)}$ ~ 10 μm

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