Lecture 14
Electrons in Magnetic Field and Fermi Surface

Classical Mechanics

Circular Motion – Cyclotron Frequency

$$\omega_c = \frac{eB}{mc}$$

$$\hbar\omega_c = \frac{e\hbar}{2mc}2B = 5.789E-5(\mathrm{eV/T}) \times 2B \approx 0.1(\mathrm{meV/T})$$

Semi-Classical Motion

Similar to classical motion, but now the orbit can be quite funky (non-circle, open, etc.) determined by the band energy

- **Closed orbit**

  Cyclotron Frequency

  $$\omega_c = \frac{eB}{m^* c} \qquad m^* = \frac{\hbar^2}{2\pi} \frac{\partial A(\varepsilon, k_z)}{\partial \varepsilon}$$

  Drude-like Hall effect (electron – and hole +!)

  $$R_H = \mp \frac{1}{nec}$$

  Saturating magnetoresistance

- **Open orbit**

  Non-standard Hall effect

  If open orbit is not parallel to current, then at high field

  - Unsaturating magnetoresistance
  - Hall angle is not 90°

Oscillatory Behavior in 1/H

- Energy, Magnetization (dHvA), Conductivity (SdH), and all other quantities
- Pure sample and low T required
- Probe of the Fermi surface

Quantum Mechanics – Landau Levels

- Energy Scale – Cyclotron Frequency

  Landau level $(n+\frac{1}{2})\hbar\omega_c$

- Length Scale – Magnetic Length – Range of Oscillation $l_B = \sqrt{\frac{\hbar c}{eB}}$

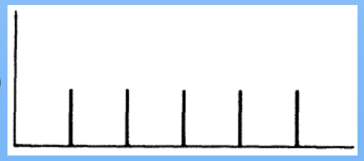
  $\vec{A} = Bx\hat{y}$ Landau gauge

  $$H = \frac{p_x^2}{2m} + \frac{1}{2}m\omega_c^2\left(x + l_B^2\frac{p_y}{\hbar}\right)^2$$

- Flux Quantum

  $$\Phi_0 \equiv hc/e = \; 4.13 \times 10^{-7}\,\text{gauss cm}^2$$

  Landau level Degeneracy $D = 2\Phi/\Phi_0$

g(ε)


Origin of Oscillatory Behavior

- Whenever N_e / D = integer, there is a discontinuity in energy.

$$N_e = Dn + D\nu \quad n = int. \quad \nu = 0...1$$

ν jumps from 1 to 0 as a periodic function of 1/H

$$Dn = N_e \quad \rightarrow \quad 1 = \Delta\left(\frac{N_e}{D}\right) \quad \rightarrow \quad \Delta\left(\frac{1}{H}\right) = \frac{1}{H_s}$$

$$D(H_s) = N_e$$

H_s: all electrons in the First Landau level

- Energy

$$E = D\sum_{i=0}^{n-1}(i + 1/2)\hbar\omega_c + D\nu(n + 1/2)\hbar\omega_c$$

Semi-Classical Point of View

- Note that $$\frac{1}{H_s} = \frac{2A}{hc/e}\frac{1}{N_e} = \frac{2\pi e}{\hbar c}\frac{1}{A_F}$$ $$A_F \equiv \pi k_F^2$$
- This can be derived from the Semi-classical Point of view

$$\frac{\hbar^2 k^2}{2m} = (n + 1/2)\hbar\omega_c$$

$$\frac{\hbar^2 A_k}{2m\pi} = (n + \frac{1}{2})\hbar\omega_c$$

$$A_k \rightarrow A_F$$

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A_F}$$

Band Structure, 3 dimensions

- From consideration of S-C EOM

$$\omega_c = \frac{eH}{m^* c} \qquad m^* = \frac{\hbar^2}{2\pi}\frac{\partial A(\varepsilon, k_z)}{\partial \varepsilon}$$

- Bohr's Correspondence Principle (ok for high n)

$$\varepsilon_{n+1} - \varepsilon_n = \hbar\omega_c$$

$$\frac{\partial A}{\partial \varepsilon} = \frac{\Delta A}{\hbar\omega_c} = \frac{2\pi}{\hbar^2} m^*$$

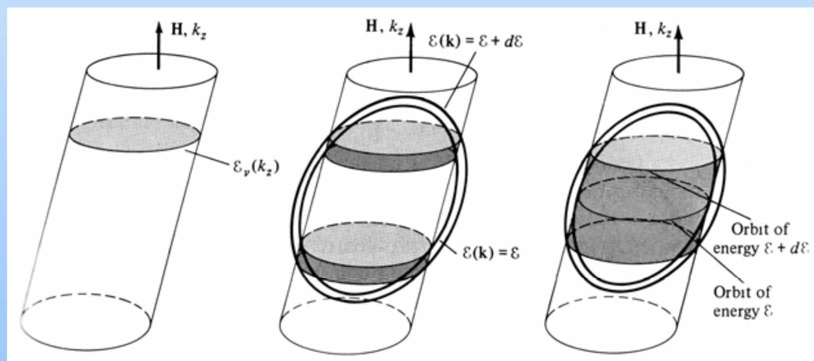
$$\Delta A = \frac{2\pi e}{\hbar c} H$$

$$A_n = (n + \lambda)\frac{2\pi e}{\hbar c} H$$

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A_F}$$

Fermi surface measuremnt

- Oscillation is due to the "popping-out" of Landau "circle" (meaning any closed orbit shape)
- In 3 dimensions, extremal orbits matter!



Landau tube with the same n

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A_{ext}}$$

- Rotate sample and measure extremal orbits – fairly complete information

dHvA, SdH – pros and cons

- Very high precision measurements (magnetic energy scale is very small)
- Requirements are often not met

long lifetime

$$\omega_c \tau \gg 1$$

low T

$$\frac{\hbar \omega_c}{k_B T} \gg 1$$

- Only E_F structure can be learned

ARPES

Angle Resolved Photoelectron Spectroscopy

- Direct; No restriction on T or τ; Not only at E_F; Powerful for correlated quasi-2,1-dim materials
- Surface Sensitive; Complicated for 3-dim materials