Lecture 8
Free Electrons

Both terms “free” and “electron” are loaded terms here.

Electrons in Solids

Binding energy below is measured relative to the chemical potential.

- **Core electrons (all solids)**

  Binding energy > ~20 eV to keV or more
- **Valence electrons (semi-conductor, metal)**

  Binding energy ~20 eV to near or at 0
- **Conduction electrons (metal, semi-conductor)**

  Binding energy 0 or small or negative (s-c)

Born Oppenheimer Approximation (Adiabatic Approximation)

Separation of Dynamics

Electron (E; $\mathbf{x}$) and lattice (L; $\mathbf{X}$) degrees of freedom

$H = H_E(\mathbf{x}) + H_L(\mathbf{X}) + H_{EL}(\mathbf{x}, \mathbf{X})$

$\Psi = \Psi_E(\mathbf{x}, \mathbf{X})\, \Psi_L(\mathbf{X})$

$\{H_E(\mathbf{x}) + H_{EL}(\mathbf{x}, \mathbf{X})\}\, \Psi_E(\mathbf{x}, \mathbf{X}) = E_E(\mathbf{X})\, \Psi_E(\mathbf{x}, \mathbf{X})$ ($\mathbf{X}$ is not a dynamic variable in this line)

$\{H_E(\mathbf{x}) + H_L(\mathbf{X}) + H_{EL}(\mathbf{x}, \mathbf{X})\}\, \Psi_E(\mathbf{x}, \mathbf{X})\, \Psi_L(\mathbf{X}) =$

$\{E_E(\mathbf{X}) + H_L(\mathbf{X})\}\, \Psi_E(\mathbf{x}, \mathbf{X})\, \Psi_L(\mathbf{X}) = E_{TOT}\, \Psi_E(\mathbf{x}, \mathbf{X})\, \Psi_L(\mathbf{X})$

Origin: vast difference between electron and ion velocities

Electrons have velocity ~ 10^8 cm/sec

Ions have velocity about 1000 times smaller

- Treat ions as static for electron problem
- Treat electrons as instantly adjusting for phonons

Breaks down in insulators or semi-metals

"Polaron" – Landau, Feynman, …; HTSC?

Drude Theory of Conductivity

- Treat Electrons as classical objects
- Collision occurs at some time scale τ
- Thought that ions can act as source of collision, i.e. mean free path ~ lattice constant – not true as we will see

$$\sigma = \frac{ne^2\tau}{m}$$

Drude conductivity=1/Drude resistivity

- With proper understanding of τ, this holds even for quantum case!

Wiedemann-Franz Law in Metals (Electron does it all – transport charge and heat)

- Thermal Conductivity

$$\kappa = \frac{1}{3} v l \frac{C_V}{V} = \frac{1}{3} v^2 \tau \frac{C_V}{V}$$

- Drude used classical gas theory to conclude that C_V ~ const (Dulong Petit) and $v^2 \sim T$, to conclude that $\frac{\kappa}{\sigma T} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$

  This happens to be correct by a factor of 2 to the experimentally correct quantum results

$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \equiv \text{Lorenz number}$$

"Universal" constant

Hall Effect

- Current flowing in the x direction.
- Magnetic field applied to the z direction.
- Hall voltage is induced in the y direction.
- Hall coefficient (~ inverse of Hall number)
  $R_H = E_y / (j_x H_z) = -1/(nec)$
  Good for some metals, but dramatically fails for some (even wrong sign!)
- Also, the magneto-resistance in the presence of H_z is the same as Drude (field indep.) – not true.