Phys 231, Fall 2007, Homework 4, due November 5

(4 points per each problem)

1. A&M 25.3
2. A&M 1.5
3. A&M 2.1
4. Very important results (2.75-77) are simply statements of particle conservation (sum rule) at different temperatures. (1) Write down the particle conservation rule between T = 0 and a finite $T << T_F$, as an integral expression of $g(\varepsilon)$ and $f(\varepsilon,\mu,T)$. [Here, the dependence of the Fermi-Dirac function on μ and T are explicitly indicated for clarity.] (2) Generally, this sum rule equation can be solved for μ iteratively. By examining leading orders of (T/T_F), re-derive (2.77). In doing so, start by re-writing the Fermi-Dirac function $f(\varepsilon,\mu,T)$ as $[f(\varepsilon,\mu,T) - f(\varepsilon,\mu,T=0)] + f(\varepsilon,\mu,T=0)$, and also considering a Taylor expansion of $g(\varepsilon)$ around μ. From the symmetry of the integral, show that $\mu(T)$ cannot have any odd power of T and that the leading term is as shown in (2.77), using $\int_0^\infty \frac{xdx}{\exp(x)+1} = \frac{\pi^2}{12}$.