



# Lecture 12

## Superconductivity

Defining characteristics of the superconductivity are the Meissner effect and the infinite conductivity, *in **that** order.*

# Meissner Effect



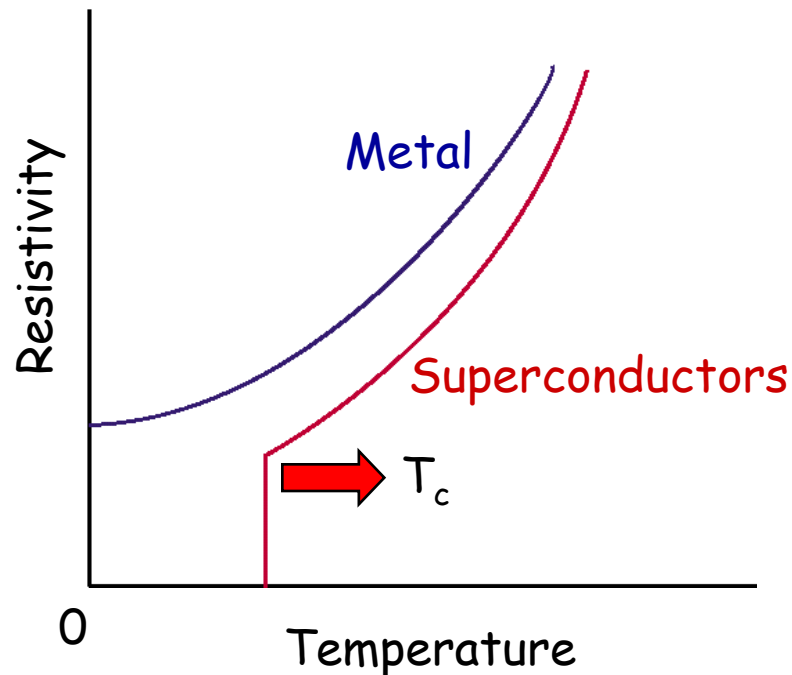
<http://www.fys.uio.no/super/levitation/>

# Essential Characteristics

**1911** K. Onnes Superconductivity in Hg

**1933** Meissner effect

## RESISTANCELESS CONDUCTION



**MEISSNER EFFECT:**  
Perfect diamagnetism

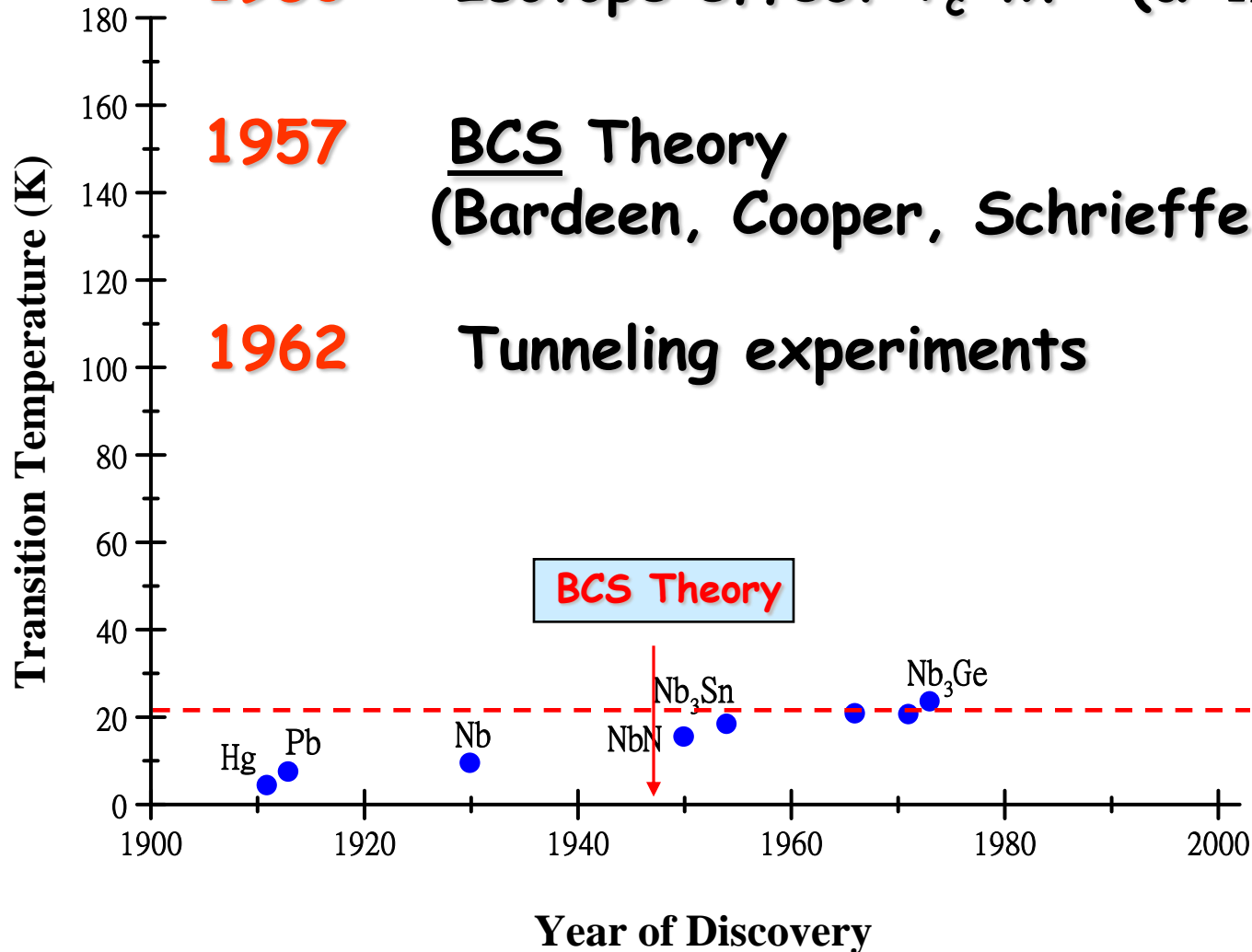


# Understanding of Superconductivity

**1950** Isotope effect  $T_c \sim M^{-\alpha}$  ( $\alpha \sim 1/2$ )

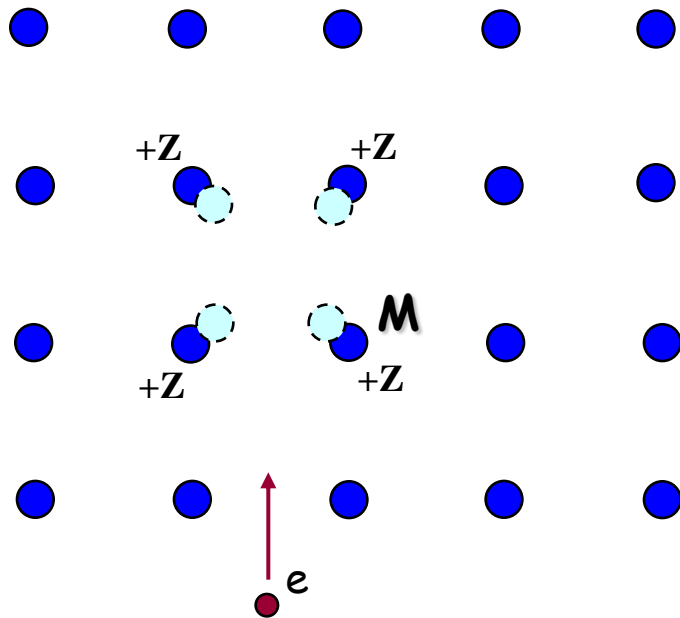
**1957** BCS Theory  
(Bardeen, Cooper, Schrieffer)

**1962** Tunneling experiments



# BCS theory

## PHONON MEDIATED PAIRING (phonon = lattice vibration)



Pairs of electrons: **Cooper pairs**

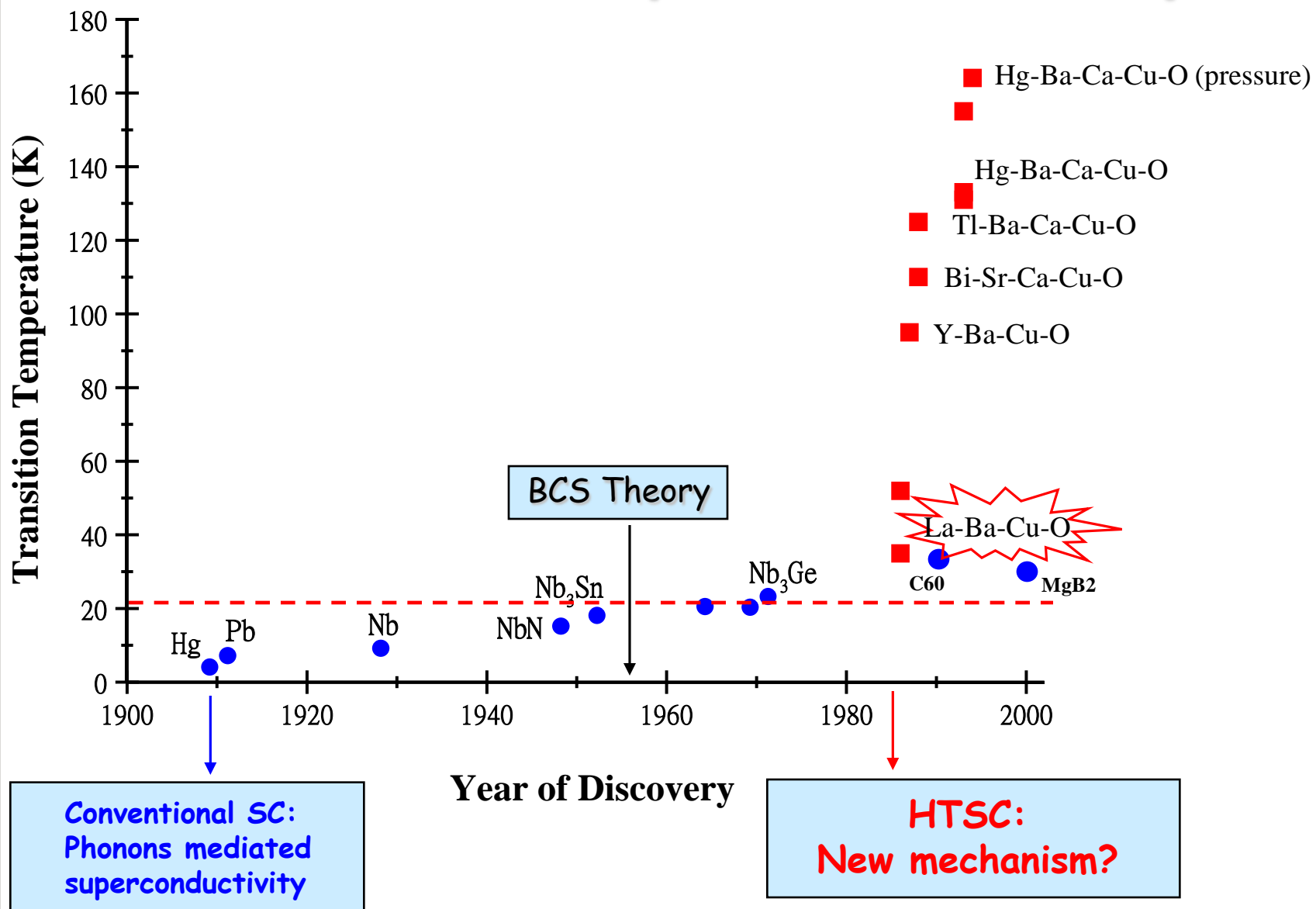
Superconducting gap:  $\Delta$

EI-ph coupling constant:  $\lambda$

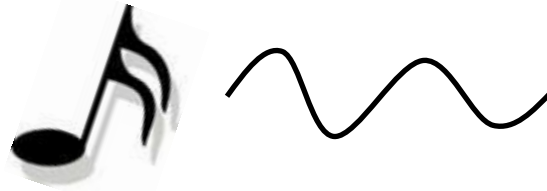
$$T_c \sim \omega_{ph} \exp(-1/\lambda) \sim M^{-1/2}$$

e-ph wins e-e at low freq. (**Slow Wins!**)

# HTSC – New Superconductivity



# Superconductivity – Dance of Electron Pairs



Origin of SC = **PAIRS** dancing to the **SAME** tune





# So How does it work?

$$\vec{j}_0 = \frac{\hbar q}{2im} (\psi_0^* \vec{\nabla} \psi_0 - (\vec{\nabla} \psi_0)^* \psi_0)$$

QM, e.g. Sakurai

$$\vec{j} = \vec{j}_0(\psi_0 \rightarrow \psi) - \frac{q^2 \vec{A}}{mc} |\psi|^2$$

$$H = \frac{1}{2m} \left( \vec{p} - \frac{q\vec{A}}{c} \right)^2 + \text{pot.}$$

(QM or H&H 7.3.1)

$\psi \approx \psi_0$  (energy gap, many-body coherence)

$$\vec{j}_0 = 0$$

$$\vec{j} = -\frac{q^2 \vec{A}}{mc} n_s$$

London Equation

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

$$q = -2e, \quad m \approx 2m_e$$

$$-\vec{\nabla}^2 \vec{A} = \frac{4\pi}{c} \vec{j} = -\frac{4\pi q^2 n_s}{mc^2} \vec{A}$$

$\lambda \sim$  a few 100 Å

$$\vec{\nabla}^2 \vec{A} = \frac{\vec{A}}{\lambda^2}$$

$$\lambda = \sqrt{\frac{mc^2}{4\pi q^2 n_s}}$$

$\vec{B}$  field is screened within the length scale  $\lambda$

Meissner Effect

Steady state No electrostat. pot.

Infinite Conductivity

$$\frac{\partial \vec{j}}{\partial t} = 0$$

$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E} = 0$$

# Two Length Scales and two types of SC

$\lambda$  = London Penetration Depth

$\xi$  = coherence length  $\sim \frac{\hbar v_F}{\Delta}$

$\Delta$ : pair binding energy

$\xi \sim$  Pair wavefunction size

Pippard non-local E & M  $\leftarrow$  due to  $\xi$

As material becomes impure,  $\lambda$  and  $\xi$  change in opposite manners.

Type 1 SC:  $\lambda(T=0) \ll \xi$

Type 2 SC:  $\lambda(T=0) \gg \xi$  (most useful)