Lecture 10
Magnetic Order

Love thy neighbors

Different Kinds of Magnetic Order

- **Ferro-magnetism** (Fe, CrO_2, EuO, etc.)
- Anti-ferromagnetism (MnO, FeF_2, HTSC cuprates, etc.)
- Ferri-magnetism (Fe_3O_4) …
- Note

1. Magnetic orders happen at fairly high temperatures (100 K ~ 1000 K) – i.e. internal field is very high due to <u>Coulomb interaction</u>
2. The above are B=0 properties, while the following designate response to finite B:
   - Para-magnetism (Curie, Van Vleck, Pauli)
   - Dia-magnetism (Larmor, Landau, London)

Interaction between Magnetic Ions

- Dipole-dipole interaction is very small

$$\vec{B} = \frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3} \quad (cgs)$$

$$\mu_B \approx 6\times10^{-9}\ eV/gauss \approx 9\times10^{-21}\ erg/gauss$$

$$B \sim \frac{\mu_B}{r^3} \sim \frac{9\times10^{-21}}{(3\mathring{A})^3} \sim \frac{10^{-21}}{3\times10^{-24}} \sim 1000\ gauss$$

$$E \sim \mu_B B \sim 6\times10^{-6}\ eV \sim 0.006\ meV \sim O(0.01)\ K$$

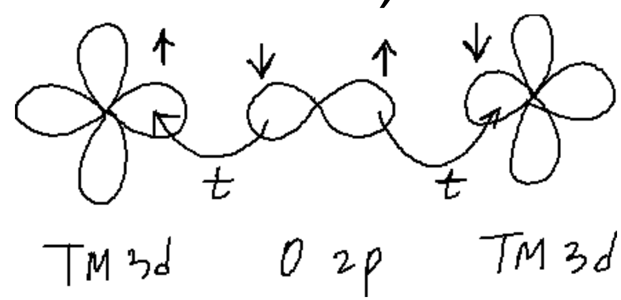
- Magnetic exchange energy results from Coulomb interactions: hopping (t) and exchange interactions
- Two most important Hamiltonians:

Heisenberg $H = -\sum_{\langle i,j\rangle} J_{ij}\, \vec{S_i}\cdot\vec{S_j}$

Hubbard H = Kinetic Energy + Exchange Coulomb Energy

Sign of J in Heisenberg Model

- Positive like "Hund's rule" (J > 0): Equal spins repel less
- **Super-Exchange** (J < 0): TM ions (such as ions of Fe, Mn, Cu etc) interact via anions



- Double exchange (J > 0), RKKY (J can be random), …

Ferromagnetism due to local spins

- Consider Heisenberg Hamiltonian with only nearest-neighbor interaction (now <i,j> means only nearest neighbor pairs, not all possible pairs) and J > 0

$$H = -J \sum_{\langle i,j \rangle} \vec{S_i} \cdot \vec{S_j}$$

- This Hamiltonian is very difficult to solve. One can however show that all spins pointing in the same direction is the lowest energy state. For instance for spin ½, it is easy to show

$$H|\psi\rangle = -NJ/4\,|\psi\rangle \qquad |\psi\rangle = |\uparrow\uparrow\uparrow\cdots\uparrow\uparrow\uparrow\rangle$$

where z direction is any random direction. (Hint: use $S_x = \frac{1}{2}(S_+ + S_-)$ $S_y = -\frac{i}{2}S_+ + \frac{i}{2}S_-$)

Ferromagnetism due to local spins

- Clearly the ground state is one in which all spins are aligned (symmetry breaking) and the excited state is in one in which spins are flipped (magnon or spin wave)
- At finite T, each spin will "fluctuate" around its mean value (even at T=0 due to quantum fluctuations)
- An elementary approach is then to ignore these fluctuations and apply a "mean field" theory

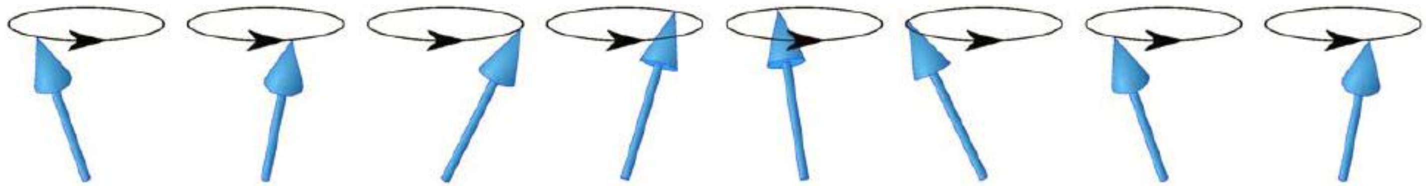
Mean field theory of FM (Weiss)

- There is an enormous internal field due to magnetization ($\lambda >> 1$): $\lambda \mu_0 \vec{M}$ $\vec{B} = \lambda \mu_0 \vec{M} + \vec{B}_{loc}$
- At high T, Curie-Weiss Law: $\chi = \frac{C}{T - T_c}$
- At T < Tc, spontaneous magentization
- T = 0: Saturation without any external field $M = N\mu$
- Provides basic picture for the FM transition but …
  - T-dependence near Tc (wrong):

    $M \propto \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$ $\chi \propto (T - T_c)^{-1}$
  - T-dependence near T = 0 (wrong):

$$M = N\mu\left(1 - 2\exp\left(-\frac{2T_c}{T}\right)\right)$$

Spin Wave and Bloch $T^{3/2}$ law in FM



http://upload.wikimedia.org/wikipedia/commons/0/01/FerromagneticMagnon.svg

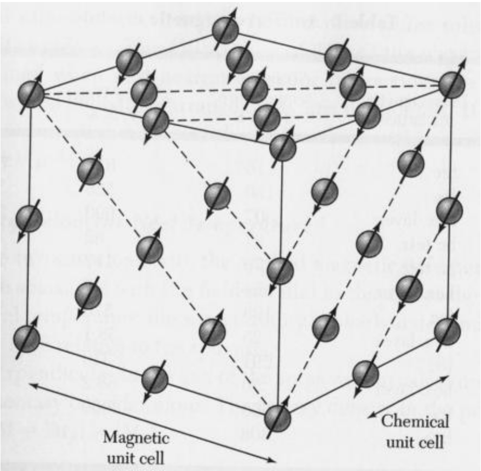
- Quantum Mechanically, spin wave means excited states with one spin flipped
- Easiest way to consider this is to do Classical Mechanics (like phonon problem)
- We know that k=0 solution must exist with energy = 0 (like the acoustic phonon case)
- FM spin wave has the property: $\omega \propto k^2$
- This leads to Bloch's law near T=0:

$C \propto T^{3/2}$ $\quad\quad$ $\vec{M}$ reduction $\sim T^{3/2}$

Antiferromagnetism

- Spins align anti-parallel to each other (J < 0 due mostly to the Anderson super-exchange)
- Different from FM in that
  - AFM state is not an eigenstate
  - Not even ground state (!)
  - But thermodynamically favored state at $T<T_N$
  - Magnetic unit cell differs from chemical unit cell
  - Spin wave is <u>really</u> like phonon:

$$\omega \approx \omega_{ex} |ka| \qquad k \to 0$$



Mn^{2+} ions in MnO (from Kittel)

High T susceptibility of FM or AFM due to local spins

$\chi \sim 1/(T - \Theta)$

(FM: $\Theta \sim O(T_C)$, <u>Curie Temperature</u>)

$\chi \sim 1/(T + \Theta)$

(AFM: $\Theta \sim O(T_N)$, <u>Néel Temperature</u>)

Θ is not exactly T_C (or T_N) because the theory presented here is very simple (only nearest neighbor interaction and mean-field)

Itinerant Ferromagnetism

- One of the mechanism for metallic FM (as in Fe)
- Due to exchange Coulomb interaction
- Exchange interaction favors spin polarization but kinetic energy favors no spin polarization **(competition)**
- The onset of the FM is given by Stoner criterion in perturbation MF theory

$$\frac{g(\varepsilon_F)}{N} \cdot U = 1$$

Stoner Ferromagnetism

- Just like Pauli Para-magnetism theory, but with internal field

$$\varepsilon_{\vec{k}\uparrow} = \varepsilon_{\vec{k}} + U n_{\downarrow} \quad \text{Exchange energy } U > 0$$

$$\varepsilon_{\vec{k}\downarrow} = \varepsilon_{\vec{k}} + U n_{\uparrow}$$

$$M = \mu_B (n_{\uparrow} - n_{\downarrow})$$

$$n = n_{\uparrow} + n_{\downarrow}$$

$n, n_{\uparrow}, n_{\downarrow}$: # densities

$$\varepsilon_{k\uparrow} = \varepsilon_{\vec{k}} + U \cdot \frac{n}{2} - U \frac{M}{2\mu_B}$$

$$\varepsilon_{k\downarrow} = \varepsilon_{\vec{k}} + U \cdot \frac{n}{2} + U \cdot \frac{M}{2\mu_B}$$

- Just like the last quiz, except self-consistency needs to be imposed (source M is the same as the result, as in Weiss MF theory) – left for reader

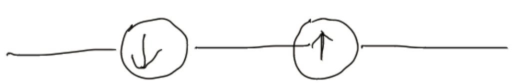
$$\frac{g(\varepsilon_F)}{N} \cdot U = 1$$

Hubbard Model and AFM

- Hubbard Model provides a basis for the Stoner FM (small U)
- Hubbard Model also provides a basis for an AFM in the opposite limit of large U

Hubbard model has an AFM when $U \to \infty$

No mixing possible



E lowered by $-\frac{t^2}{U}$

Also, Mott-Hubbard Insulator
(cf. Midterm #2 and #4(e))
I.e., **AFM MH Insulator**