



# Lecture 9

## Magnetism

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Magnetism is probably the most complicated phenomenon in condensed matter physics. Main driving forces are Spin and Coulomb interaction.

# Origin of Magnetic Moment in Solids

- Electrons are the main source  
(Nuclear spins do come in handy as in NMR)
- Magnetic moment
  - $\boldsymbol{\mu} = -\mu_B (\mathbf{L} + g_0 \mathbf{S}), g_0 = 2.0023\dots \approx 2$
  - $\mu_B = e\hbar/(2m)$ 
    - $\mu_B = 9.27e-24$  J/T (SI) or  $5.8e-9$  eV/gauss
    - For nucleons, reduction by  $\sim 2000$
    - Remember, B field  $\sim 100$  T =  $1e6$  gauss is the strongest field that one can generate, so magnetic perturbation energies are quite small.

# Some Atomic Physics ...

- **Spin-orbit coupling:**

- $H_{LS} = \mathbf{L} \cdot \mathbf{S} \frac{e^2}{r^3 8\pi\epsilon_0 m^2 c^2}$
- Goes like  $\sim \alpha^2 \times$  unperturbed energy
- The heavier the atom, the greater  $\langle H_{LS} \rangle$
- The closer the orbital is to the atom, the greater  $\langle H_{LS} \rangle$
- Quite important for the band structure of semiconductors (44 meV for Si, 0.34 eV for GaAs)

- **Zeeman interaction:**

- $H_B = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu_B (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}$

- For very light atoms (Li, e.g.), the Zeeman term can be more important than the spin-orbit coupling.

- **For most solid problems, the spin-orbit coupling is more important than the Zeeman term.**

- Treat spin-orbit first, and then, treat Zeeman as a perturbation.
- Good Quantum Numbers:  $J, L, S, J_z$
- $\langle H_B \rangle = g(L, S, J) \mu_B J_z B$  (**g: Lande g-factor**; eq. 7.10; see Sakurai)

$$\mu_{\text{eff}} = g \mu_B J_z$$

# Hund's Rule for Atomic Spectra

- Empirical rule that works pretty well for 3d Transition metal (TM) and 4f rare earth (RE) ions
- Many electrons in an un-filled orbital
  - First rule: maximize S
  - Second rule: maximize L
  - Third rule:  $J = |L-S|$  if less than half full and  $J = L+S$  if more than half full
  - Notation:  $^{2s+1}L_J$
- E.g.,  $V^{3+} : 3d^2 \ ^3F_2$ ,  $Fe^{2+} : 3d^6 \ ^5D_4$ ,  $Mn^{2+} : 3d^5 \ ^6S_{5/2}$
- Roughly, the physics is due to Coulomb interaction and spin-orbit interaction

# Curie Paramagnetism

- Localized magnetic moments in response to external B field
- $M = \chi H$
- Curie law at high T, small B:  $\chi = C/T$ 
  - $C = N p^2 \mu_B^2 \mu_0 / (3 k_B)$  (N is volume density)
  - $p = g [J(J+1)]^{1/2}$
- Saturation at low T, large B:  $M = Ng\mu_B J$
- p agrees well for RE, but not so for TM
- For TM ions,  $p = 2 [S(S+1)]^{1/2}$  works much better!?



# Quenching of Orbital Moment in Transition Metal Ions in Crystal

- Empirically observed (table 7.2)
- Effect of Crystal Field (large  $\sim 1$  eV in TM ions)
- $L_z$  is not a good quantum number
- $\langle L_z \rangle$  (also for x,y) is often zero

# Pauli Para-magnetism of Metals

- Response of Fermi sea to the magnetic field
- Spin-up electron's energy goes up and spin-down ... down, both by  $\mu_B B$
- $M = \mu_B(n_{\uparrow} - n_{\downarrow}) = \mu_B^2 g(E_F) B$  (g is per volume, but for both spins)
- $\chi = M / H \approx \mu_0 \mu_B^2 g(E_F)$  (T independent)
- $\chi \sim \mu_0 \mu_B^2 N / E_F$  (N is volume density), i.e. weakened by  $T/T_F$  (quantum behavior – only those at  $E_F$  respond) compared to Curie susceptibility (classical behavior – all spins respond)

# Diamagnetism

- Lenz's law
- Landau Diamagnetism
  - Orbital Motion of Electrons
  - For free electron,  $\chi_{\text{Landau}} = -\chi_{\text{Pauli}}/3$
  - For effective mass  $m^*$ ,  $\chi_{\text{Landau}} / \chi_{\text{Pauli}} \sim (m/m^*)^2$   
i.e.  $\chi_{\text{Landau}} \gg \chi_{\text{Pauli}}$  (semi-cond) and  $\chi_{\text{Landau}} \ll \chi_{\text{Pauli}}$  (renormalized metal)
- Larmor (Langevin) Diamagnetism due to closed shell ions ( $\mathbf{A} = -\mathbf{r} \times \mathbf{B}/2$ ,  $H = (\mathbf{p} + e\mathbf{A})^2/2m$ , The last term of  $H \sim r^2 B^2$ )
- For non-magnetic materials, often it is necessary to consider all susceptibilities (Pauli, Landau, Larmor, and Curie – often due to magnetic impurities)