Phys 155, Winter 2007, Homework 2, due 5pm, Jan 31

(Each problem is 4 points. For problems from the textbook, please note that solutions are included in the textbook. However, use those solutions simply as guide, if necessary. If you do, your answers should show enough details that reflect your own understanding.)

1. Problem 2.1 of H&H

2. (a) Show that equation (2.7) reduces to the following continuum elastic wave equation, if the wave length is much longer than the lattice constant. [Note that under this condition the lattice constant can be taken as an infinitesimal, dx, of the differential calculus.] Verify that v is the same as (2.13).
$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$
(b) Generalize this solution to the case of diatomic chain, (2.15) and (2.16), and show that the same form of wave equation is obtained with a correct velocity for the wave, as given in page 44, i.e. $v = a\sqrt(\frac{K}{2(M+m)})$, where a is the lattice constant (not the distance between adjacent atoms, which is $a/2$).

3. [Peierl's transition] Problem 2.2 of H&H with a modification at the end: When plotting dispersion curves, plot these two cases together (i) K_1=K_2=K (this should be the same as the mon-atomic chain except for the periodicity doubling), (ii) when K_1 and K_2 differ. In the case of (i), indicate where the two branches – acoustic and optical – meet.

4. (a) Solve the Debye model mathematically for high temperature and low temperature limits of E, C, N_{ph}, as we did in class, when the spatial dimension is 2, instead of 3. (b) For the general case when the phonon dispersion relationship is $\omega \propto k^{\alpha}$ $(\alpha > 0)$, and the spatial dimension is D, make a table, like the one we made in class, for high temperature and low temperature solutions of E, C, N_{ph}. That is, express these quantities in terms of N (number of primitive lattice points or number of atoms, if you like), $\theta_D, T, k_B, \alpha,$ and D, ignoring numerical factors. Solutions should be obtained without using any integrals, but by relying on physical arguments alone, as in class. You can use the following integrals for part (a): $\int_0^\infty dx\, x^2 \frac{1}{e^x-1} = 2.404 \ldots \equiv 2c_A$ and $\int_0^\infty dx\, x \frac{1}{e^x-1} = \frac{\pi^2}{6}$.

5. Problem 2.6 of H&H with an addition: Using the Heisenberg uncertainty principle, estimate the uncertainty of the atomic position, Δx, associated with the zero point energy. Express Δx as a fraction of the Ar-Ar distance = 3.72Å.