Notes for Lecture 10

Spherical and plane mirrors

10.1 Mirror equations

Both spherical and plane mirrors are described by the following mirror equations, assuming that the object is small or, conversely, the mirror is large, so that the angle of incidence can be considered very small.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{10.1}$$

$$m = \frac{d_i}{d_o} = -\frac{h_i}{h_o} \qquad \text{laternal magnification} \tag{10.2}$$

$$f = \frac{r}{2} \qquad \text{imaging point of a distant object} \tag{10.3}$$

I recommend using these equations and a ray diagram using at least two rays.

In all of these treatments, we assume that the mirror is perfect, in which case the specular reflection, $\theta_i = \theta_f$, that follows the law of reflection is the only reflection to consider. For a real mirror, the surface is never perfect, and there will be loss of intensity to diffuse background.

Two types of spherical mirrors exist: concave ($r, f > 0$) and convex ($r, f < 0$). The rear view mirror or the side view mirror of a car is a convex mirror, while a circular vanity mirror that can be often found in a hotel room is a concave mirror. For a convex mirror, $|m| < 1$ always (prove it!): in other words, you get a view with a small mirror, which is exactly what you need for a rear view mirror or a side view mirror. Also, it always gives you a virtual upright image: again, this is a quite reasonable thing to have for mirrors to use in a car. For a concave mirror, you can get a real inverted image or a virtual upright image, depending on where the object is placed.

Especially important is the fact when the object is close to the focal point, you get great magnification. Slightly inward past the focal point is the object position for which a vanity mirror can be very useful—large magnification and upright virtual image. Through the focal point, the nature of the image flips: i.e., if the object is placed inside of the focal point, you get a virtual upright image, while if the object is placed outside of the focal point, you get a real inverted image. Please check out these types of mirrors and try to feel their curvatures, next time you see them!

For a plane mirror, $r \to \pm\infty$, + for concave and − for convex, gives the correct set of equations. Now, why *should* we take $r \to \pm\infty$ limit to get a plane mirror? You can think about this way—we feel that we are on a plane when we know that we are on a round planet, all the time. Why? Because the radius of the Earth is so large compared to our every life length scales, we have no trouble perceiving everyday objects to be on a flat piece of land—the approximation of a small arc of an infinitely large circle as a line is as real as the fact that only the zero length arc is flat is flat for any small circle.

10.2 Virtual or real?

Generally, "virtual" means mathematically possible but not realized. We already had some discussion of it in the context of the mathematical view of the superposition principle. Virtual means possible but not realized.

The same definition applies here as well, actually. In optics, we speak of real object or virtual object, or real image or virtual image. Here goes the definition and the rationale.

First of all, let us note the following basic point: all real objects that are visible are visible because they reflect or emit light—every point of such an object is the source for an outgoing spherical wave.

Now, suppose that an object is placed in front of a mirror—that is a real object. Suppose that an object is placed in the back of a mirror. Clearly such an object is a simply hidden object—it is an object of no concern to us. However, consider a more exotic situation, where somehow light impinges on a mirror in a converging fashion, so that if you locate the convergent point, then that point is in the back of a mirror. This is the case of a virtual object. As we shall see, it is possible to create a virtual object using multiple optical elements. So, a virtual object is an object that the input light converges to on the "wrong side" (back side) of the mirror, while a real object is an object that the input light diverges from on the "right side" (front side) of the mirror. Here, the input light refers to the light incident on the mirror.

Similarly, a real image is the image onto which the output light converges to, while a virtual image is the image from which the output light diverges from. Here, the output light refers to the light reflected from the mirror. Note that a real image is on the "right side" (front side) of the mirror, while a virtual image is on the "wrong side" (back side) of the mirror.

As with the general definition of the word "virtual" note that in the above discussion there is no light field around the virtual object case or the virtual image.

10.3 Mirror equations and graphs

The mirror equation $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$ can be rewritten as

$$d_i = f + \frac{f^2}{d_o - f} \tag{10.4}$$

Students of optics must be able to plot the *function* $d_i(d_o)$, i.e. d_i as a function of d_o, as a graph, when the range of d_o is given and the value of f is given. If you do this for a convex mirror ($f < 0$) with a real object ($d_o > 0$), then you should find that d_i starts from 0 (for $d_o = 0$) and steadily decreases to f as $d_o \to \infty$. Also, you should be able to prove that, for $f < 0$ and $d_o > 0$, $|m| < 1$: so a convex mirror always de-magnifies.