Notes for Lecture 8

Beats, Doppler effect

8.1 Beats

Beats occur when two sounds with similar frequencies are superposed. Let us take

$$D_1(x,t) = A\sin(k_1 x - \omega_1 t + \phi_1) \tag{8.1}$$
$$D_2(x,t) = A\sin(k_2 x - \omega_2 t + \phi_2) \tag{8.2}$$

Using $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$, the superposed wave becomes

$$D = D_1 + D_2 = 2A\sin(k_a x - \omega_a t + \phi_a)\cos(k_b x - \omega_b t + \phi_b), \tag{8.3}$$
$$X_a = \frac{X_1 + X_2}{2}, \qquad X = k, \omega, \phi \tag{8.4}$$
$$X_b = \frac{X_1 - X_2}{2}. \qquad X = k, \omega, \phi \tag{8.5}$$

Assuming that $\omega_1 \approx \omega_2$, we see that $\omega_a \approx \omega_1 \approx \omega_2$ and $k_a \approx k_1 \approx k_2$ (since $k_1 = \omega_1/v, k_2 = \omega_2/v$). So, the sine function that occurs in the expression for D is just like one of the two original waves, up to a phase shift, to a good approximation.

The superposed wave can be written as

$$D(x,t) = A_B(t)\,\sin(k_a x - \omega_a t + \phi_a), \tag{8.6}$$
$$A_B(t) = 2A\cos(k_b x - \omega_b t + \phi_b). \tag{8.7}$$

The key observation is that $A_B(t)$ is a more slowly varying function than $\sin(k_a x - \omega_a t + \phi_a)$. Due to this, at any instant, one might hear the original frequency of the sound, but slowly the intensity of the sound that one hears will go up and down, following the time dependence of $A_B(t)^2$ (recall from LN 6 that the intensity is proportional to A^2). This is the so-called **beat frequency**. $|A_B(t)|$ is often referred to as the envelope function—the wave $D(x,t)$ can be understood as the fast wave that is enclosed in the envelope function, which is slowly varying wave (see Fig. T16.30(b)).

Note that $A_B(t)$ has frequency ω_b. However, A_B^2 (or $|A_B|$) has half the period, and so it has double the frequency—this is the beat frequency by definition, since the beat frequency is the frequency that one hears, i.e., the frequency of the intensity.

$$\omega_B = 2|\omega_b| = |\omega_1 - \omega_2|. \qquad \textbf{beat frequency} \tag{8.8}$$

8.2 Coherent waves

Just like the word "phase," the word "coherent" is a big deal, and it deserves much of your attention.

Example T16-12

One thing to note is that in this example the two loud speakers are assumed to give out sounds that are precisely the same: the phase constants ϕ for the two waves are exactly the same. This is the tacit assumption of this example, due to, e.g., the two speakers hooked up to the same sound source. Such sources of waves, like these two loud speakers, are called **coherence sources**.

As in this example, if two wave sources are placed at different locations but are emitting the same sound wave, then we can write each wave as

$$D_1 = A\sin(kx_1 - \omega t + \phi_1), \tag{8.9}$$
$$D_2 = A\sin(kx_2 - \omega t + \phi_1), \tag{8.10}$$

where x_1 is the distance to wave source 1 (speaker 1) and x_2 is the distance to wave source 2 (speaker 2). Note that the initial phases are identical (ϕ_1), which means that D_1 and D_2 are coherent waves.

How about incoherent waves? An example would be two light bulbs that are emitting light. While typical light bulbs emit light of many frequencies, we may focus our attention on only one frequency out of all frequencies emitted by the bulbs. Also, we assume that the two light bulbs emit light at the same intensity (so the same A's). For lights coming from two different (consumer-grade) light bulbs, we may observe that the two wave trains coming off at one moment have certain phase relationship

(ϕ_1 and ϕ_2 are constants where $D_1 = A\sin(kx_1 - \omega t + \phi_1)$ and $D_2 = A\sin(kx_2 - \omega t + \phi_2)$) but that the difference between ϕ_1 and ϕ_2 is randomly changing from one moment to another. The result is that on average there is no fixed phase relationship between D_1 and D_2. This defines the so-called incoherence.

Now, a question. What if the same sound wave is fed into two loud speakers but with one sound that is shifted in phase by a certain set amount.

$$D_1 = A\sin(kx_1 - \omega t + \phi_1), \tag{8.11}$$
$$D_2 = A\sin(kx_2 - \omega t + \phi_2), \tag{8.12}$$

where

$$\phi_2 - \phi_1 = \text{constant}. \tag{8.13}$$

Are these two waves coherent waves? The answer is yes. They are perfectly coherent waves. Coherence does not necessarily mean that the two initial phases are the same ($\phi_1 = \phi_2$); the only requirement is that the difference between the two initial phases remain constant ($\phi_2 - \phi_1$ = constant).

An easy way to empirically test the coherence of two waves is to see if they produce interference patterns. The intensity maxima and minima result due to constructive interference and destructive interference, respectively, only if there is some degree of coherence between two wave sources. For completely incoherent waves, the two phases are completely random with respect to each other and so there cannot be any interference patterns.

8.3 Doppler effect

Here, some comments on the Doppler effect are presented. The textbook must be read carefully to understand the origin of the Doppler effect (change of the wave length if the source is moving; change of the wave speed if the observer is moving).

Relativity?

Consider two situations: (1) a source of sound moving with speed v_r towards a stationary observer, and (2) an observer of sound moving with the same speed v_r towards a stationary source.

In case (1), the observed frequency is $f_o = f\frac{v}{v-v_r}$, while in case (2), the observed frequency is $f_o = f\frac{v+v_r}{v}$. Here, v is the normal speed of sound. While these two expressions are practically the same if $v_r/v \ll 1$, they do differ if v_r/v is not small. The question is "why this behavior?".

According to the relativity principle, there is no such thing as an absolute velocity, and so only relative velocities are meaningful. If you think in this way, you might led to believe that the two cases (1) and (2) *must* be equivalent to each other, since the relative velocity of the source to the observer is the same! But, wait, there is a problem. Not only do the source and the observer exist, but also there is air. In case (1), the air is (assumed to be) stationary. While in case (2), *in the observer's reference frame*, the air is moving! Since f_o is the frequency perceived by the observer, this difference is significant, and makes the two cases different.

This highlights an important point: **the Doppler effect formula is derived for the medium at rest**. In general, if one applies it in cases when the medium is moving, it will give an erroneous result.

Now, if $v_r/v \ll 1$, then the difference of the two cases due to the condition of air can be ignored, which is the reason why the two Doppler effects give practically the same result. This is no longer true if v_r/v is not small.

However, in the case of light in free space, there *is no medium* other than vacuum. So, our reasoning above shows that the relativity principle must hold! Indeed, the Doppler effect of light in vacuum depends only on the relative velocity of the source and the observer: the above cases (1) and (2) would be completely equivalent for light propagating in vacuum.

Example T16-15

In this example, the Doppler shift for the echo of a sound that bounces off a moving target is measured by the very person, who sent out the original sound. Please follow the solution given during the lecture, or in the book. Note that this problem involves two legs of motion: the first leg from the time that the sound goes off and arrives at the surface of the moving object, and the second leg from the time that the reflected sound goes off from the surface to the original person. In the first leg of the motion of the sound wave, this person is the sound source, while in the second leg of the motion of the sound wave, this person is the observer. In the first leg of the motion, the surface, at which the sound wave is reflected is the observer, while in the second leg of the motion, the surface is the sound source.

This example reminds me of a speed gun that a high way patrol uses to catch speeding cars! Indeed, the Doppler effect is what such a speed gun is based on; however, it is the Doppler effect of light (radio wave), not sound. **The Doppler**

effect of light is given by a completely different formula! (And, it is given in the textbook, if you like to see it.)