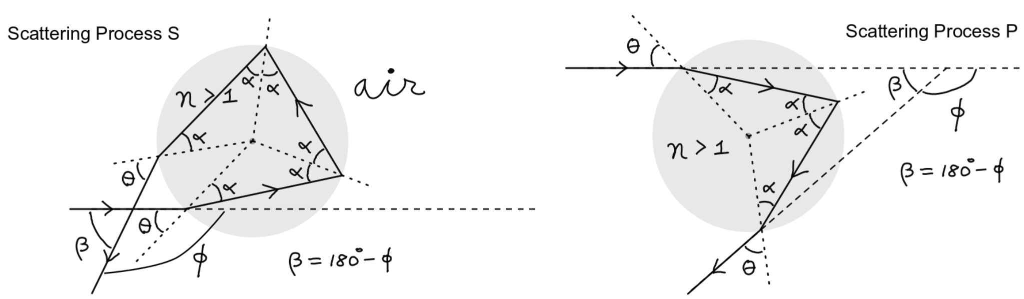
Due Feb. 26, Thursday

All problems must be solved symbolically first. Then, any numerical answer, when required, can be computed by substituting numbers into the symbolic expression at/near the very end. Solving problems symbolically means deriving the answer in terms of symbols, instead of numerical values. All problem numbers refer to those in the textbook. (Not all problems may be graded in detail, due to limited man power; however, you must do all problems.)

For each problem, you are required to use sensible symbols, by defining or adopting them yourself, for your symbolic solution. If you are unsure how to do so, feel free to ask (or look back at homework 1)!

Problem 1 (10 points) Problem 32.54 (Snell's law; prism; dispersion).

Problem 2 (50 points = 30 points + 20 extra credit points) [Rainbow] Consider a sphere made of material with index of refraction $n > 1$ (e.g., a water drop hanging in air). A ray of light hits the sphere. The incoming ray of light propagates in the horizontal direction. It then scatters with the sphere to emerge, bent by the angle ϕ with respect to the initial direction. This is the "scattering angle" ϕ. ϕ and or its complementary angle β is of our utmost concern.



Two important scattering processes are depicted above. Scattering process S is what we will consider in this problem. Scattering process P is shown only as a reference. In either case, $0 \le \theta \le \frac{\pi}{2}$.

(a) Using Snell's law and the law of reflection, find the function $\phi = \phi(\theta, n)$ for scattering process S. Make sure that $\phi \to 0$ (forward scattering) as $\theta \to 0$.

(b) Show that $f(\theta) = \phi(\theta, n)$ has a maximum as a function of θ, if $n \le 3$, at

$$\theta_m = \sin^{-1}\sqrt{\frac{9 - n^2}{8}}, \quad 0 < \theta_m < 90^\circ,$$

for the given range of $\theta:$ $0 \le \theta \le 90^\circ$. [Hint: $\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$.]

(c) That the function $f(\theta) = \phi(\theta, n)$ has an extremum at θ_m is an important fact, for the following reason. It is possible to show that the intensity of light that appears between angles ϕ and $\phi + \Delta\phi$ (for small $\Delta\phi$) is given by $G(\theta)\Delta\theta \approx G(\theta)\Delta\phi / \left|\frac{df}{d\theta}\right|$, where $G(\theta)$ is a regular (i.e., non-divergent) function of θ. Note that $\left|\frac{df}{d\theta}\right|$ vanishes at an extremum point of $f(\theta)$, giving rise to a *divergent* intensity at $\theta = \theta_m$. This means a singular intensity maximum at $\theta = \theta_m$ or at the corresponding $\phi_m = \phi(\theta = \theta_m)$. So, this is the reason why the extremum angle ϕ_m, or $\beta_m = 180° - \phi_m$, is singularly important. Now, we shall assume water droplets only. Under ambient conditions, the refractive index (n) of water varies monotonically from 1.330 (red) to 1.343 (violet) for the visible spectrum of light. Calculate the values of $\beta_m \equiv 180° - \phi_m$ for red light and violet light, and thus show that the values of β_m for the visible spectrum of light span about 3 degrees, with maximum β_m for red and minimum β_m for violet—this explains the secondary arc of a rainbow.

(d) Explain why in the secondary arc of a rainbow, the colors are reversed compared to the primary (main) arc. Explain also why the region between the primary arc and the secondary arc is dark.

(e) Now, consider a two refraction scattering event (no internal reflection), which is a more likely event than the two scattering events depicted above. Show that in this case, there is no extremum for ϕ as a function of θ.

Problem 3 (10 points) Problem 32.74 (Snell's law).

Problem 4 (10 points) Problem 32.83 (Total internal reflection).

Problem 5 (10 points) Problem 33.10 (Lens).

Problem 6 (10 points) Problem 33.20 (Lens, combination).

Problem 7 (10 points) Problem 33.98 (Glasses).

Problem 8 (10 points) On the ground, there is a nearly flat mirror, which is, in fact, a concave mirror with the radius of curvature 2.4 m.

(a) On day 1, a bird is observed to be flying directly above the mirror at 1.0 m height from the mirror. Find the location and the size of the image, and find whether the image is virtual or real and whether it is inverted or not.

(b) On day 2, the bird is observed at the same location. However, it rained during the previous night, and now a thin layer of water ($n = 1.33$) has formed above the mirror. Find the location and the size of the image of the bird, and find whether the image is virtual or real and whether it is inverted or not.