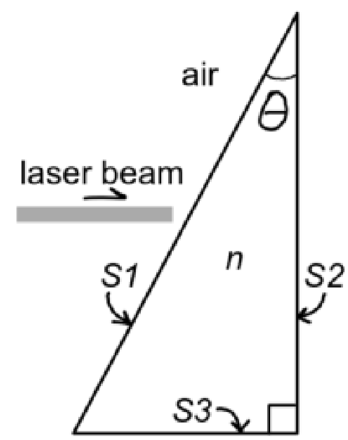
1 A prism has a vertex angle of 55° and an index of refraction of 1.45. A ray of light enters one side of the prism at angle of incidence, θ. As θ is varied from 0 to 90°, there is a range of θ values for which no light emerge from the opposite side of the prism. Find that range.

Your solution:

2 You have been put in charge of designing and installing a prism, whose cross section is a right traingle with a certain fixed top vertex angle θ (e.g., 30 degrees or 60 degrees), as shown below. The incoming light is a thin cylindrical "pencil beam" from a laser. The beam comes in horizontally, i.e., parallel to the basal plane (*S3*) of the prism. The index of refraction n for the prism material is at your disposal, and your task is to make sure that no light leaks through the side (*S2*): i.e., after the laser beam enters the prism through *S1*, it must go through a total internal reflection at *S2*. Find the minimum index of refraction (n) required, as a function of θ. [Notes: Only symbolic answer is required. No numerical answer will be graded. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$, $\sin^2 x + \cos^2 x = 1$.]

Your solution:



3 One morning, Ray realizes that the far points of both his eyes have decreased (to 50 cm) and the near points have decreased (to 18 cm). He goes to an optometrist, who confirms this finding and gets a pair of glasses made for Ray. When Ray wears his new glasses, his glasses sit 2.0 cm in front of his eyes.

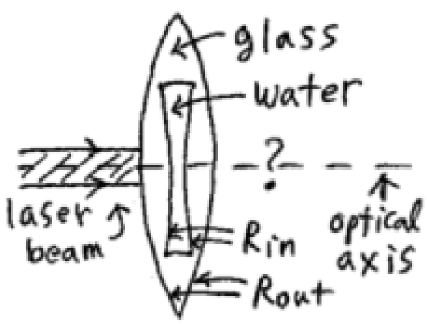
(a) What should be the prescription (in D, diopters) for the glass lens (for either eye), assuming it has restored the normal far point for Ray?

(b) What is Ray's new effective near point with this corrective lens?

(c) One lens surface of the corrective lens is convex with the radius of curvature 10 m. Determine whether the other lens surface of the corrective lens is convex or concave and what its radius of curvature is. Assume that the index of refraction for the lens material is 1.5.

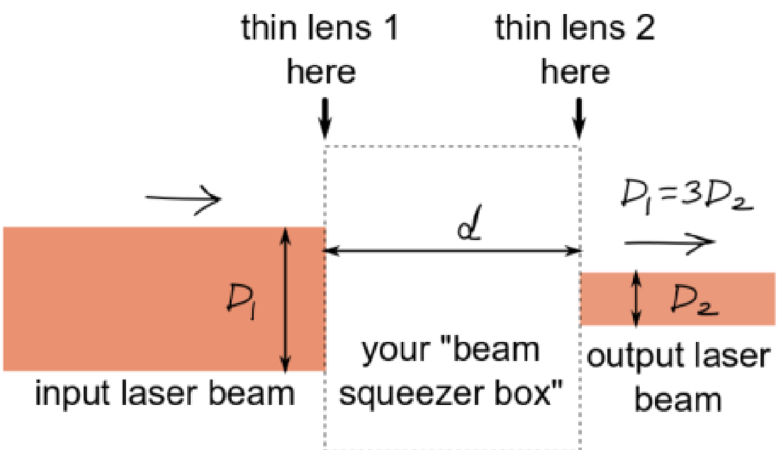
A thin converging lens made of glass ($n_g = 1.55$) happens to have an inclusion of water ($n_w = 1.33$) at the center. The lens surfaces have radii of curvature $R_{out} = r_1 = r_2 = 10$ m, and the water inclusion has the shape of a diverging lens with radii of curvature $R_{in} = r_1 = r_2 = -20$ m. The optical axis of the water inclusion is the same as the optical axis of the glass lens. A laser beam of 10 mm diameter hits the center of this lens from the left, parallel to the optical axis. Assume that the diameters of the lens and the water inclusion are (much) greater than 10 mm. (a) Find the position along the optical axis, where the laser light gets focused to a point. Find this position as a function of symbols n_g, n_w, R_{out}, and R_{in}, first, and then find its numerical value (as usual). (b) Is the point calculated in (a) a real image/focus or a virtual image/focus? (c) A screen is placed perpendicular to the optical axis, on the right side of the lens, at distance 20 m away from the lens. Find the diameter of the laser light when it hits this screen. [Consider the initial laser beam as a perfect cylindrical beam and *ignore any diffraction*. Lensmaker's equation: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$.]

Your solution:



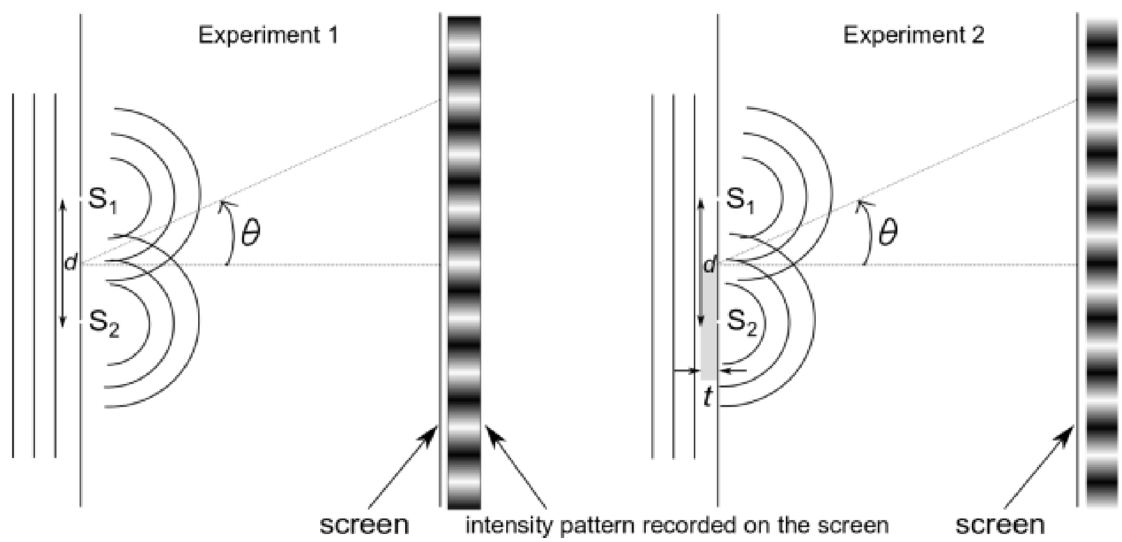
5 A laser beam is in the shape of a cylindrical "pencil beam." You are asked to "squeeze the beam" while preserving the cylindrical shape. After thinking for a minute, drawing on your excellent knowledge of optics gained from 5B, you say "Sure thing, boss, I can do that!" You are given a converging lens with a certain fixed focal length $f_C > 0$. You must order an additional lens and place the two lenses at a distance d, as shown below, to accomplish the goal of reducing the diameter of the beam to **one third** of the original diameter D_1 (D_1 is much smaller than the lateral dimension of either lens). **(a)** Find f_1 (focal length of lens 1), f_2 (focal length of lens 2), and d (lens-to-lens distance) as a function of f_C. Your answers, including the signs of f_1 and f_2, must be explained clearly and concisely. **(b)** Is the image of thin lens 1 real or virtual? **(c)** Is the object of thin lens 2 real or virtual? [Hints: Ray diagram should help. Ignore any diffraction. One of f_1 and f_2 must be equal to f_C.]

Your solution:



6 Young's double slit experiment is carried out with slit spacing d (e.g., 1 mm; slit width ≈ 0) and light of wave length λ (e.g., 500 nm). Initially (Experiment 1), the intensity at $\theta = 0$ is the maximum intensity. You are then asked to make bright and dark fringes change places, by placing a highly transparent slab of thickness t and index of refraction n at the back of one of the two slits (Experiment 2). Find the **minimum** value of t that will make this happen, as a function of (some) parameters defined above. Only the symbolic solution is required. [Notes: The schematic diagram below in no way hints the magnitude of t correctly, nor the distance from slit to screen. You do *not* need to consider reflections caused by the slab, as it is highly transparent ($n > 1$, but $n \approx 1$). Only symbolic answer is required. <u>No numerical answer will be graded.</u>]

Your solution:



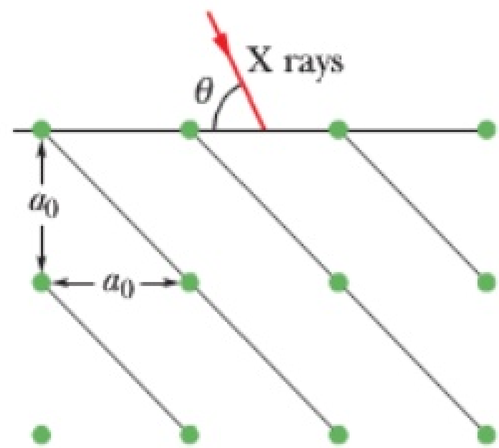
A soap bubble 250 nm thick is illuminated by white light. The index of refraction of the soap film is 1.36. Which colors are not seen in the reflected light? Which colors appear strong in the reflected light? What color does the soap film appear at normal incidence?

8

A diffraction grating is used to monochromatize ultra-violet light. Specifically, it needs to resolve 120.00 nm and 120.10 nm wave length lights **in the second order**. The ruling (= groove) density is given by 5000 per cm. The initial beam is incident on the grating at normal angle. (a) At what angle would the second order light for 120.00 nm appear? (b) What is the minimum number of rulings that the grating must have for the required resolution? (c) What is the minimum number of rulings that the grating must have, if the requirement changes to resolving 120.00 nm light from 120.05 nm light? (d) What is the relative intensity of the principal maximum peak of case (c) in comparison to that of case (b), assuming the minimum number of rulings in each case? Assume all other conditions are the same.

9

In this figure, first-order reflection from the reflection planes shown occurs when an x-ray beam of wavelength 0.260 nm makes an angle $\theta = 63.8°$ with the top face of the crystal. What is the unit cell size a_0?



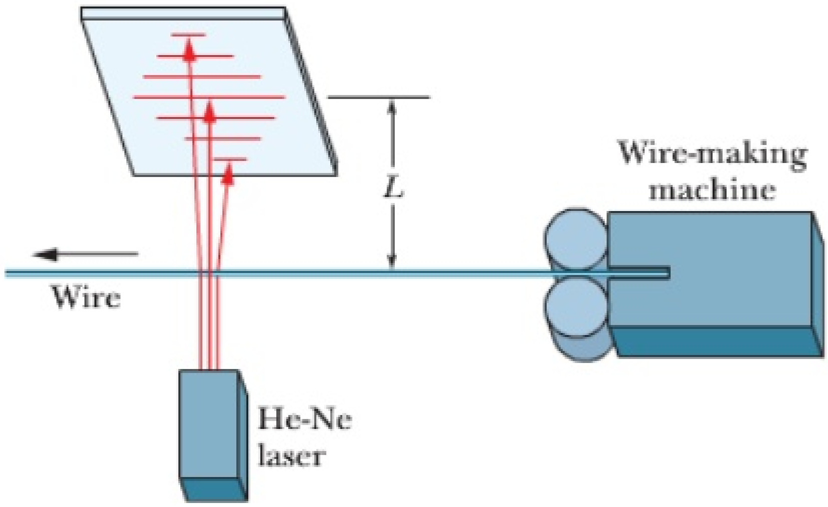
The *resolving power* of a microscope is defined by RP$= f\theta$, where f is the focal length of the objective lens and θ is the minimal angular separation of two resolvable objects according to Rayleigh's criterion.

Yellow light of wavelength 589 nm is used to view an object under a microscope. The diameter of the objective lens is 5 mm and its focal length is 9 mm.

(a) Compute the resolving power of the microscope.

(b) Suppose water (with an index of refraction $n = 4/3$) fills the space between the object and the objective lens. How is the resolving power of the microscope changed?

11 Manufacturers of wire (and other objects of small dimension) sometimes use a laser to continually monitor the thickness of the product. The wire intercepts the laser beam, producing a diffraction pattern like that of a single slit of the same width as the wire diameter (Fig. 36-37). Suppose a helium–neon laser, of wavelength 632.8 nm, illuminates a wire, and the diffraction pattern appears on a screen at distance $L = 2.60$ m. If the desired wire diameter is 1.37 mm, what is the observed distance between the two tenth-order minima (one on each side of the central maximum)?



It is said that an eagle can spot a rabbit from one mile ($\equiv l$) away, i.e., it can resolve $d \approx 20$ cm, the typical size of rabbit's big features, at distance l.

(a) Find the diameter, ϕ, of the pupil of an eagle eye, assuming that the wavelength of the light is given by λ and the finite resolution of eagle's eye arises solely due to diffraction.

(b) A person's eye is able to resolve 0.1 mm ($\equiv d_p$) at 25 cm ($\equiv l_p$) away. If this person is to have the same eye sight as an eagle, with the help of a telescope, what is the angular magnification M required for the telescope?

Only symbolic answers in terms of l, d, λ, l_p, and d_p are required. No numerical answers will be graded. You can also leave out the sign for M.