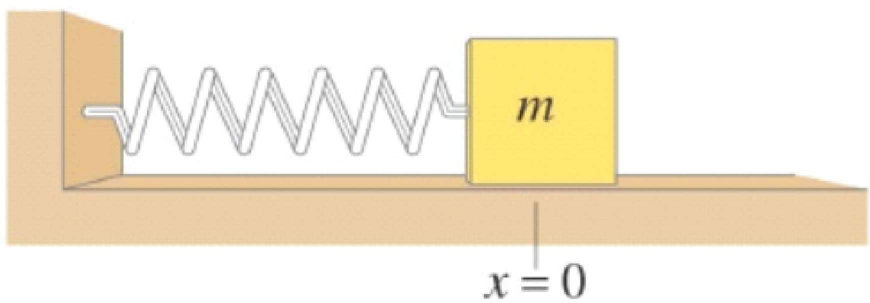
1 A block with a mass of $m = 0.5$ kg is attached to a spring with a spring constant $k = 50\,\mathrm{N/m}$. Let x be the displacement from the equilibrium position. You may neglect the mass of the spring. At equilibrium, the center-of-mass of the block is located at $x = 0$, as shown in the figure below.



Suppose that at time $t = 0$, the block has its maximum speed of $20\,\mathrm{m/s}$ and is moving to the left.

(a) Write down an equation for the displacement of the center-of-mass of the block as a function of time, $x(t)$. Determine the numerical values of the amplitude and the angular frequency of the oscillation.

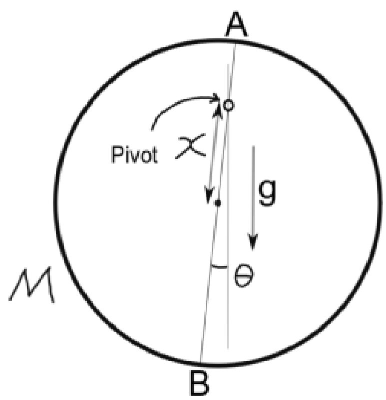
(b) Find the minimum time interval required for the center-of-mass of the block to move from its location at $t = 0$ to a point 1 m to the left of its equilibrium point.

(c) For what values of x is the potential energy three times the kinetic energy?

HINT: Use the conservation of energy to write the square of the velocity v^2 directly in terms of x^2 by comparing the total energy at any point of the oscillation with the total energy at the point of maximal displacement. You can then immediately solve for x (without ever specifying the time t).

2 Inside a hoop with mass M and radius R, a thin massless stiff wire is mounted across the diameter AB (see the diagram below). The hoop oscillates in a vertical plane around a pivot point, which is on the stiff wire at distance x away from the center ($0 < x < R$). (a) Find the period of small oscillation ($|\theta| \ll 1$). (b) An extremum period, T_m, occurs at $x = x_m$, $0 < x_m < R$. Find x_m, T_m, and the nature (minimum or maximum) of the extremum. [Hint: the moment of inertia for the hoop around its center of mass is given by MR^2.]

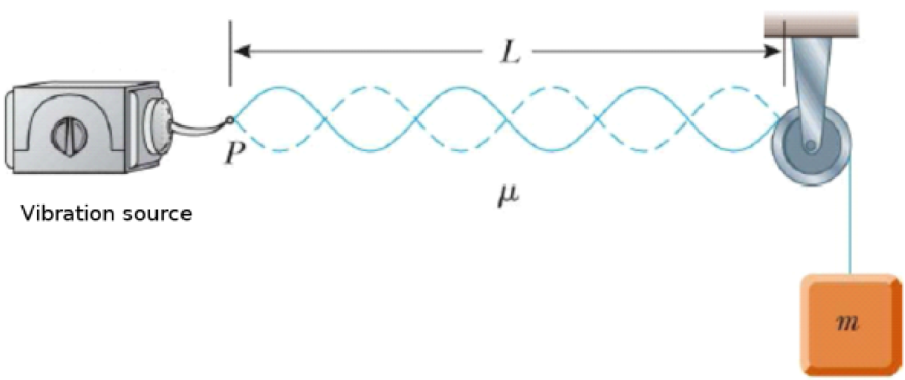
Your solution:



3

A block of mass m is hung from a string, with a linear mass density $\mu = 3\text{g/m}$, that passes over a fixed massless pulley. The string is connected to a machine ("vibration source"), which generates a small vibration at the tip, with constant frequency f. The distance from the pulley to the machine is given by $L = 3$ m. When the mass m is either 16 kg or 25 kg, standing wave is observed. Point P corresponds to a node for standing wave to a good approximation. No standing waves are observed for any mass between these two values. [Note: the shape of the standing wave shown below is for illustration only, and does not necessarily correspond to any of the masses given in the problem.] (a) Find the frequency f of the machine. (b) Find the mass of the heaviest block for which standing wave could be observed.

Your solution:



4

A stone is dropped into a dark well whose depth is not known. You hear the sound of the stone hitting the bottom of the well t seconds after you drop it. Find the depth of the well, d, as a function of t, the speed of sound in air v (= 343 m /s), and the surface gravity g (= 9.80 m/s^2). Make a table for the values of d for t = 1.00 sec, 5.00 sec, and 10.0 sec, and compare them with the values for $\frac{1}{2}gt^2$.

Your solution:

A policeman in a stationary car measures the speed of approaching cars by means of an ultrasonic device that emits a sound with a frequency of 41.2 kHz. A car is approaching him at a speed of 33.0 m/s. The wave is reflected by the car and interferes with the emitted sound producing beats. Find the frequency of the beats. The speed of sound in air is 330 m/s.

6

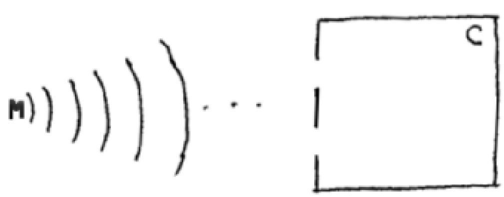
In a playground, there are many children running about. At any given random time, each child is making a sound of 1.0 kHz, is running at speed 1.0×10^1 m/s in a completely random direction. You are a stationary observer of sound. Find the "Dopper broadening" ratio $\Delta f/f$, where f is the mean frequency (1.0 kHz), and Δf is the width of the distribution of frequencies that you hear. Here, we shall simply take Δf as the difference between the maximum possible frequency that you can hear and the minimum possible frequency that you can hear. Take the speed of sound to be 340 m/s.

Your solution:

7

A child is playing in a corner of a room of size 10.00 m × 10.00 m. The room has two completely open doors on a wall, opposite to one of the two walls, in whose shared corner the child is playing. The two doors are separated by 3.00 m (center-to-center), symmetrically positioned so that each door is 3.50 m from each corner that the wall connects to. Mother calls for the child from outside the room, generating a sound of frequency 640 Hz. Mother is standing at equal distance, 20.00 m, from each door, when she does so. Her voice is loud enough so that it is easily audible at 40 m away. The child never responds. Mother walks into the room, and demands explanation for no response. The child says to have never heard the call. Is the child's claim valid? Take the speed of sound to be 340 m/s and the width of door to be 0.70 m.

Your solution:



Notes: The last number for the width of the door is to be understood as small enough to be ignored (it can be set to "zero," if you like). It is an unimportant number in this case.
Hint: the two open doors act as coherent sources of sound.

8

A 150 g air hockey puck glides at v_1 = 2.30 m/s towards another stationary 200 g puck, head-on. The sides of pucks are very sticky (or have Velcros) and, thus, the two pucks undergo a completely inelastic collision to become one body in a time interval of 7 ms. A person suggests that roughly half the decrease in kinetic energy of the two-puck system is transferred to the environment by sound during the collision. You are asked to be a judge to figure out if this idea makes sense. To evaluate this idea, find the implied sound level in decibel at a position 0.8 m from the pucks. What does your answer suggest about the above idea? [$I_0 = 10^{-12}$ W/m^2 corresponds to 0 decibel. Typical sound levels: normal city traffic sound corresponds to 80 dB, while very loud sound at threshold of pain (causing possible ear damage) corresponds to 120 dB. "Just in case hint": the momentum conservation holds during the collision.]

Your solution: