Notes for Lecture 19

Statics+

Statics is very easy, but often *seems* very difficult. If one remembers the following points, then it can be very easy.

1. Equilibrium conditions: net force = 0, net torque = 0.

2. Do not assume that you know forces, unless you really do. Figure them out from the equilibrium conditions. Forces that you are expected to know are simple ones such as gravity. You are supposed to know the directions of forces such as normal force and tension force; but, you are (almost) never expected to know their magnitudes, you need to figure them out. For some problems, the tension or compression nature of force may not be clear and must not be pre-assumed.

3. For the net torque, it can be measured from any reference point (see Section 19.1 for why). So, choose the reference point (i.e., the origin) to be the most difficult point, e.g., the point at which many forces apply at the same time, or the point at which the most unknown force applies.

4. Be very clear with Newton's 3rd law. That is, know which body you are considering for the free body diagram, and remember to flip the direction of the contact force if you consider the free body diagram for an adjacent body.

19.1 Torque

In considering statics, the following is an important point.

If net force is zero, and if one shows that the net torque is zero around one point, then the net torque is zero around any point.

Let us see how this comes out.

Assume that there are N forces $\vec{F}_i$ $(i = 1, ..., N)$ acting on the body in question at N positions $\vec{r}_i$. The net torque is given by $\vec{\tau}_{net} = \sum_i \vec{r}_i \times \vec{F}_i$. By definition, this is the torque around the origin of our coordinate system, in which vectors $\vec{r}_i$'s are defined. Now, let us consider a fixed position vector $\vec{a}$. Let us say that we like to calculate the torque around $\vec{a}$, not around the origin. The position vector $\vec{r}_i = \vec{a} + \vec{r}\,'_i$, where $\vec{r}\,'_i$ is the position vector relative to $\vec{a}$. Thus, we get

$$\begin{aligned}\vec{\tau}_{net} &= \sum_i \vec{r}_i \times \vec{F}_i \\ &= \sum_i (\vec{a} + \vec{r}\,'_i) \times \vec{F}_i \\ &= \vec{a} \times \sum_i \vec{F}_i + \sum_i \vec{r}\,'_i \times \vec{F}_i \\ &= \vec{a} \times \vec{F}_{net} + \vec{\tau}\,'_{net} \qquad (19.1)\end{aligned}$$

where $\vec{\tau}\,'_{net}$ is the net torque around $\vec{a}$. **Note that $\vec{\tau}_{net} = \vec{\tau}\,'_{net}$ if $\vec{F}_{net} = 0$!** This proves our assertion above.

19.2 Vector product

In the above discussion, we used the notion of the vector product between two vectors. If $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ and $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$, then $\vec{A} \times \vec{B}$ is defined as

$$\vec{A} \times \vec{B} \equiv (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z} \qquad (19.2)$$

The following can be proved.

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta \qquad (19.3)$$

where θ is the angle between the two vectors. Geometrically, this means that the magnitude of a vector product between two vectors is the area of the parallelogram formed by the two vectors. What is the direction of the vector product? It follows the **right hand rule** or **right handed screw rule**: rotate $\vec{A}$ towards $\vec{B}$, and figure out which was the screw will translate, if this rotation was that of a right handed screw – that is the direction of the vector product. Note that, then, the direction of the vector product is always perpendicular to both $\vec{A}$ and $\vec{B}$. I invite you to prove this property algebraically, by using the above definition of the vector product in terms of components.

19.3 Torque and center of gravity

In Section 19.1, we said that a force applies at a certain position. This is not ambiguous at all for a small infinitesimal volume of mass. It is always a good idea to divide a big object into many small infinitesimal volume of small masses, if it is not clear where an external force applies. However, it would be vastly more convenient if we can somehow think that the total amount of force acts at a single point. *This is not always possible.*

If this is possible in the following sense,

$$\vec{\tau}_{net,g} = \sum_i \vec{r}_i \times \vec{F}_{i,g} \qquad \text{always valid} \tag{19.4}$$

$$= \vec{R}_{CG} \times \sum_i \vec{F}_{i,g} = \vec{R}_{CG} \times \vec{F}_{net,g} \qquad \text{not always possible!} \tag{19.5}$$

then we would have successfully defined the **center of gravity** $\vec{R}_{CG}$ of our body in question. Here, $\vec{r}_i$ is the position vector for an infinitesimal mass element of our body, $\vec{F}_{i,g}$ is the gravitation force on that mass element, $\vec{F}_{net,g}$ is the total gravitational force (subscript g), and $\vec{\tau}_{net,g}$ is the total gravitational torque. One way to see why the above identification of $\vec{R}_{CG}$ is not always possible is to note that it is necessary that $\vec{\tau}_{net,g}$ must be perpendicular to $\vec{F}_{net,g}$, but it is easy to note that this cannot be satisfied in a general field.

A notable case when the center of gravity is well defined is when the gravitational field is constant. In this case, the center of gravity is the same as the center of mass.

19.4 Something extra

As we close this lecture, let us note something extra for fun.

Heisenberg uncertainty principle

Let us consider the single slit diffraction again and discuss it in terms of this famous principle, valid for all kinds of waves, not just for a quantum particle. On the other hand, what we call "light" (that is, classical light) is really just a lot of what we call "photons" (that is, quantum particle), and so the Heisenberg principle is expected to be valid no matter how you look at light (as classical wave or as quantum particle). The more you squeeze the beam, the more precise the position of light will become. It then follows from the uncertainty principle that the momentum of light becomes more uncertain, in other words its momentum values become more broadly distributed. In this example, we are talking about the y position and the momentum along the y direction, p_y, only, taking the y axis as the axis parallel to the width of the slit. Note that, initially, right out of the slit, the beam does not really have any sense of movement along the y direction. However, the uncertainty principle requires that p_y, while averages to zero, will have a broad distribution of finite values. Importantly, the width of this distribution, Δp_y, is inversely proportional to $\Delta y \approx D$. A large value of Δp_y means a faster spreading in the y direction, while the mean position stays the same ($y = 0$). So, this can be understood as the reason why the beam expands faster and produces a bigger diffraction spot, when D is made smaller! Of course, this would be valid for all waves that we considered so far, e.g., the sound wave. So, the Heisenberg uncertainty principle is not just for quantum mechanics.

For those of you who know $\hbar$, here is some more discussion. Indeed, here is the precise quantum mechanical meaning of $D \sin\theta \sim \lambda$ in terms of Δy and Δp_y. $\Delta y \sim D$. $\Delta p_y \sim \hbar k \sin\theta = \hbar \frac{2\pi}{\lambda} \sin\theta$. So, $D \sin\theta / \lambda \sim \Delta y \Delta p_y / (2\pi\hbar)$. So, $D \sin\theta \sim \lambda$ means $\Delta y \Delta p_y \sim 2\pi\hbar$, which represents the Heisenberg uncertainty principle!