Notes for Lecture 18

Pressure

The study of fluids is eased greatly if one has a certain level of understanding of pressure. So, let us delve a bit into this important phenomenon.

18.1 Pressure – definition

One general definition of pressure is force per area. In particular, pressure refers to force *normal to* surface per the area of that surface[1]. Please read the book for the unit and so on. The following conversion formulae for often used pressure units are convenient to remember: 1 atm = 1.013 $\times 10^5$ Pa = 1.013 bar = 1013 mbar = 760 mm Hg = 760 Torr = 14.7 psi. The SI unit is Pa (Pascal) = N/m^2.

Pressure is a scalar quantity. From the above definition, this fact seems plausible since it involves an inner product of the force with the surface normal vector ($P = -\vec{F} \cdot \vec{n}/A$, where $\vec{n}$ is the surface normal of the surface on which the pressure is measured). However, this is not sufficient. More discussion about the scalar nature of pressure is given below, in Section 18.5.

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[1] Force parallel to surface per unit area is called (shear) stress. A general stress includes both pressure and shear stress.

18.2 Pascal's principle and Archimedes' principle

These two amazing principles were established well before physicists came to learn what pressure really means in terms of its microscopic origin. How were they established? By the only way any principles are established, before the full scope of things are understood much later—by experiments. This is more a rule than an exception, of course. Major laws or principles are established to explain phenomena, and we know their true meanings after decades or centuries.

The details of these two principles can be read about in the textbook, and they are not repeated here. You can even find the "proof" of the Archimedes's principle. This proof is very cute and very fine (and we went over it in class). You must study it carefully.

However, for truly understanding both these principles, the underlying assumptions may be the most important to understand properly.

The principal assumption in both principles is that the pressure that we are interested in is the so called **equilibrium pressure** or **static pressure**. The equilibrium pressure is the pressure that a fluid (or any substance) has when it is in a "**thermal equilibrium**" state.

What is a thermal equilibrium state? This topic will not be treated in depth here, since it deserves more formal treatment in your later courses; however, some potentially useful words are offered. A thermal equilibrium state is a state in which there is no macroscopic motion of any kind (no current, no angular momentum, no momentum) and the system is characterized by a definite volume, a definite density, and a definite temperature. In everyday terms, it is a still state, whose apparent (read, macroscopic) properties are time independent. Such a state is easy to notice. A cup of water that has been on a table a few minutes would be quite still no matter what the initial condition for the water was, and if this is the case, then it can be regarded as in thermal equilibrium. The notion of thermal equilibrium turns out to be very useful even if the system that we consider is *not* in thermal equilibrium. How so? For instance, consider ocean water. Such a large body of water has certain distributions of temperature, density (albeit nearly constant due to the near incompressibility of water), and pressure, not to mention the fact that it has ocean currents. So, a large body of water generally cannot be considered as an equilibrium state, except in some crude approximation. However, if a small volume of water around a point in space is considered, then that volume can be considered as a equilibrium state, to a very good approximation, as long as any existing currents are small [2], since the

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[2] In the current context, this assumption means that the speed associated with a current is much smaller than the average speed of molecules in equilibrium. Since the latter is very fast (on the order

temperature, pressure and density are nearly constant. When a system is not in a thermal equilibrium as a whole, but it can be divided into many small parts, which are in thermal equilibrium to a good approximation, we refer to the system as being in a **local equilibrium** state, a state that applies to many realistic situations. In contrast, a system which is truly in a thermal equilibrium state by itself is referred to as bing in a **global equilibrium** state. Note that all possible local equilibrium states include global equilibrium states as well.

Both Pascal's principle and Archimedes' principle must be viewed as being concerned with a system that is in a local equilibrium state. This is also true for Bernoulli's equation. In all of these cases, **pressure refers to the equilibrium pressure that can be defined precisely in a local region**.

18.3 Bernoulli's principle

This principle can be derived from the work-energy theorem, assuming non-viscous (i.e., no friction) laminar flow (i.e., non-turbulent flow) of an incompressible fluid. Please study the textbook derivation, which uses the definition of pressure as the normal force per area. Since we do not assume friction, the system is conservative, and the work-energy theorem leads to the energy conservation. The resultant Bernoulli equation reads

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{conserved (constant)} \tag{18.1}$$

To interpret this equation, let us consider a unit volume of fluid. We consider this a small system, and call it S. We can always make this assumption come true, by taking the unit of volume to be very small! Note that the mass of S is given by ρ. What the above equation says is this: as this system S moves from one part of the fluid to another part, the sum of its pressure and its total mechanical energy ($\rho g y + \frac{1}{2}\rho v^2$) is conserved.

This "total" mechanical energy is *not* really the total energy of the fluid. Why? It is because microscopically a fluid is not a dead motionless state. It is quite the opposite—molecules are moving very fast, on the order of the speed of sound, and they are bumping into one another like crazy, resulting in very chaotic random motions. These motions clearly contribute to an "internal energy." Now, looking at the above conservation equation, it is very tempting to identify P as this "internal energy" per volume—because, then, the above Bernoulli equation is just an energy conservation

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the speed of sound or higher), this small current assumption tends to be easily valid as a rule.

equation! Indeed, you will learn later (in thermal physics or a later course of 5) that this *is* the correct picture if underlying assumptions are properly understood[3].

The pressure in the above equation is the equilibrium pressure. It is the pressure felt by a bug riding the fluid. If the flow is contained within a tube, then this is also the same pressure that the tube wall feels. However, P is *not* the pressure that you will feel if you put your hand and interrupt the flow. That will be P plus the pressure due to the net momentum of the fluid. Notice that the fluid that moves is *not* in equilibrium. However, a bug riding with the fluid will not know this, and will see the fluid as being in equilibrium.

18.4 Pressure and normal force

Suppose you divide a fluid into two parts with a planar boundary, and ask the question, what kind of force acts between these two parts on this boundary plane? Generally, the answer is a repulsive force. Part A repels part B, and part B repels part A. Of course, these two forces are equal and opposite forces, satisfying Newton's 3rd law. Note some similarity of this situation with the normal force that one encounters in simple Newtonian mechanics problems involving solids. Normal forces are exchanged between two solid surfaces, when two surfaces are in contact and a pressure is applied (either indirectly by gravity, for a horizontal boundary plane, or by squeezing or pressing, for a boundary plane in any orientation). This is a so-called compression force. Put another way, when two fluids meet, they naturally try to compress each other due to the pressure that each fluid naturally exerts on the other.

18.5 Pressure in a fluid is a scalar quantity

While the similarity of the normal force and the force due to pressure is nice to notice, we must know which concepts to use in what situations and what differences there are between these concepts.

Generally, the normal force refers to the force at an interface between two solids. If a solid is immersed in a fluid, then we talk about pressures, rather than normal

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[3] More precisely speaking, the following is true. The process assumed for Bernoulli equation is the so-called "adiabatic process," where there is no heat exchange between systems such as S with other similar systems. In this case, one can show that P represents the so-called "enthalpy," which is a measure of the internal energy, per unit volume. You might have heard about enthalpy in chemistry. If not, you definitely will learn it in thermal physics.

forces on the surfaces of the solid, as this is a more appropriate thing to do. Note that normal forces on a solid propagates along the direction in which the force is applied in the first place[4]. The pressure applied to fluids is quite different. It applies in all directions.

There are some mundane ways in which we can convince ourselves that the pressure applies in any direction, in a fluid, and its value is independent of direction. Our palm, for example, is a fine device to measure pressure. The fact that when you orient your palm in air in any direction without feeling any difference in pressure means that the pressure is indeed constant independent of direction. Another example is the radial inflation or the radial deflation of the balloon.

What is the microscopic explanation for this? It is based on what is called the "kinetic theory" of fluid. What is essential in this microscopic picture is that a fluid consists of molecules that are (more or less) free to move except when they encounter one another. The result is a state of many many molecules moving at very high speed in random directions at any point in time.

So, pressure at a point in a fluid is the same independent of direction, and this is the reason why pressure is a scalar quantity.

How does this pressure propagation occur in a fluid? It is through the chain of local equilibria established over the length scale of the mean free path[5] of molecules. So, a fluid in equilibrium can be thought of as many many volumes of micro fluids, each of which is in local equilibrium and has the dimension of the mean free path. Of course, the local equilibrium state can be much bigger in dimension than the mean free path. If the fluid is free and in equilibrium state, then, the whole fluid is in equilibrium, since each local equilibrium state looks identical. If the fluid is in an external field (like gravitational field), local equilibria will look slightly different along the direction of the force. However, if the change of state is small, then we can still talk about a local equilibrium state that is much larger than the mean free path. For instance, a cup of water is to a good approximation an equilibrium state, even though its height is much larger than the mean free path of water molecules. This is because the pressure and the density differences from the top to the bottom are negligibly small, even though they are changing. Likewise, any still air mass near the face of the earth a few meters high can be considered to be in a local equilibrium. However, an air mass that is a km high cannot be considered to be in an equilibrium state; it can be thought of as consisting of successive layers of small thickness, each

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[4] If a solid is confined in a tight volume, and is pressed on one side, then all sides will exert pressure. However, unlike fluid, these pressures will not be equal in general. They will be dependent on which faces we look at. There is no Pascal's law for solids.

[5] This important length scale can range typically from $O(0.1)$ nm to $O(100)$ nm, the latter more typically for gas.

of which is in a local equilibrium.

Note, however, that a local equilibrium cannot be established at length scale smaller than the mean free path.

The establishment of small local equilibrium volumes over the length scale of the mean free path and the isotropic mutual influence of such small volumes (isotropic due to the fluid nature) throughout the fluid are the microscopic reasons for Pascal's law.

It is important to note that molecules that exert pressure on any surface present come from the length scales of their mean free path. So, this is why the pressure is fundamentally a local phenomenon.