# Notes for Lecture 16

# Diffraction

## 16.1 Diffraction from a rectangular slit

The thing about diffraction is that it occurs everywhere. Looking up dictionaries, I learn that the word diffraction drives from a Latin word meaning “shattering, breaking apart.”

How do we “break apart” a beam of light? Oddly enough, what we need to do is very simple—just make the beam small by using a mask! In a single slit diffraction experiment, the diffraction occurs since the slit plate masks the majority of light, and the beam profile right after the slit plate looks just as we “ordered”—the cross section of the beam looks just like the slit that we made light go through. Are there any other thing that we do to light than simply masking the beam when we place a slit plate on a beam of light? Sure, depending on what the slit material is we might be doing some additional thing such as causing some light to reflect off the inner edge of the slit. However, clearly such effect can be minimized if we use a weakly reflecting material, and we shall thus ignore such a side effect. So, the role of a slit plate is masking an incoming beam of light, so that its cross section is of a certain shape that we like.

To be quantitative, it helps to consider a simple rectangular slit of dimensions $D \times L$. We will often call $D$ the width of the slit. $L$ is the other side (“height”) of the rectangle. We take the coordinate system such that $D$ is along the $y$ axis, and the slit plate and the screen are assumed to be parallel to each other (as in our double slit setup), and both are perpendicular to the $x$ axis. So, in this setup, the parameter $l$ (the distance between the slit plate and the screen) is an $x$ coordinate value.

In this coordinate system, the center of the slit is considered as the origin, and the $L$ side of the slit is along the $z$ axis. We shall focus attention to the $xy$ plane only ($z = 0$). We shall also assume that $L$ is either very small or very large. In either case, we do not need to consider the effect of $L$ to figure out the pattern at $z = 0$ (proving this statement is left for your exercise).

With this standard setup, we can prove that (cf., LN 15)

$$D\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots \quad \text{(Destructive interference)} \tag{16.1}$$

$$D\sin\theta = 0 \quad \text{(i.e., } \theta = 0\text{)} \qquad \underline{\text{Maximum constructive}} \text{ interference} \tag{16.2}$$

$$D\sin\theta \approx \pm\left(m + \frac{1}{2}\right)\lambda \qquad m = 1, 2, \ldots \quad \text{(Other minor) constructive interference)} \tag{16.3}$$

Note that the "other minor" constructive interferences, indicated by the last equation give quite weak peaks (cf. Example 35-3 of the book) compared to the central peak due to the maximum constructive interference condition. Note also that the central peak, which occurs between $m = \pm 1$ destructive interferences, is about twice larger than the other maximum peaks, which occur about half way between two destructive interference conditions corresponding to two adjacent $m$ values with the *same* sign. LN 15 has a detailed mathematical expression (Eq. 15.35) from which the above formulas can be derived.

The result is that most intensity falls into the central peak which is twice wide as other peaks of intensity. Therefore, the first zero intensity condition

$$D\sin\theta = \lambda \tag{16.4}$$

is important, since it provides the size of the central peak.

## 16.2 Diffraction from a circular aperture

What if the single slit is a circle? Then, the above calculation breaks down, and the solution involves a slightly more complicated function (Bessel function). The resulting diffraction pattern is known as the **Airy pattern**, whose first minimum occurs at

$$D\sin\theta = 1.22\lambda. \tag{16.5}$$

Note that this minimum condition is just like the above for the rectangular slit case, except for the factor 1.22.

For small $\theta$, we get, using $\sin\theta \approx \theta$, and

$$\theta \;\approx\; 1.22\frac{\lambda}{D}. \qquad (16.6)$$

According to the Rayleigh criterion, this is the minimum angular separation of distant objects required for them to be resolved.

**The more you squeeze, the quicker it will expand.**

Let us think about what the equation $D\sin\theta \sim \lambda$ means in terms of how much you "squeeze" the beam of light, and how much the beam diffracts, i.e. how large it becomes on the screen, responding to this squeezing. The smaller $D$ is, the more we "squeeze" the beam. The larger the value of $|\theta|$ (with an upper limit of $\pi/2$, of course, enforced by the setup), the larger the diffraction spot, since $y = l\tan\theta$ will become larger in magnitude when $|\theta|$ increases. From the above equations, $D\sin\theta \sim \lambda$, it is clear that the more you squeeze the beam (small $D$), the more it diffracts (large $\theta$). Lastly, note that we did not really "squeeze" the beam, since we simply masked out the majority of light and reduced the cross section of the beam to a small size. However, the physics of diffraction is independent of the incoming intensity of the beam, and so the same diffraction phenomenon will occur also, *if* we somehow squeezed a large cylindrical beam of light to a small cylindrical beam of light, which we can really do using two lenses!

Finally, couple of points. (1) We do *not* really need a slit to observe diffraction. As long as we can impart the same initial condition (as the beam's initial condition just after it comes out of the slit) *somehow* using other means than using a slit, then the same diffraction effect will occur! (2) If we have an "anti-slit," e.g., a circular object blocking the light, diffraction of light will occur in a similar manner. In particular, the center of the screen will be a local maximum in intensity.