

Notes for Lecture 9

Doppler effect, Light

9.1 Doppler effect

What can be found in the textbook is omitted, regarding the Doppler effect. Instead, just some comments are presented.

Relativity?

Consider two situations: (1) a source of sound moving with speed v_r towards a stationary observer, and (2) an observer of sound moving with the same speed v_r towards a stationary source.

In case (1), the observed frequency is $f_o = f \frac{v}{v-v_r}$, while in case (2), the observed frequency is $f_o = f \frac{v+v_r}{v}$. Here, v is the normal speed of sound. While these two expressions are practically the same if $v_r/v \ll 1$, they do differ if v_r/v is not small. The question is “why this behavior?”.

According to the relativity principle, there is no such thing as an absolute velocity, and so only relative velocities are meaningful. If you think in this way, you might led to believe that the two cases (1) and (2) *must* be equivalent to each other, since the relative velocity of the source to the observer is the same! But, wait, there is a problem. Not only do the source and the observer exist, but also there is air. In case (1), the air is (assumed to be) stationary. While in case (2), *in the observer's reference frame*, the air is moving! Since f_o is the frequency perceived by the observer, this difference is significant, and makes the two cases different.

This highlights an important point: **the Doppler effect formula is derived for the medium at rest**. In general, if one applies it in cases when the medium is moving, it will give erroneous result.

Now, if $v_r/v \ll 1$, then the difference of the two cases due to the condition of air can be ignored, which is the reason why the two Doppler effects give practically the same result. This is no longer true if v_r/v is not small.

However, in the case of light in free space, there *is no medium* other than vacuum. So, our reasoning above shows that the relativity principle must hold! Indeed, the Doppler effect of light in vacuum depends only on the relative velocity of the source and the observer: the above cases (1) and (2) would be completely equivalent for light propagating in vacuum.

Example T16-12

We discussed this example. One thing to note is that in this example the two loud speakers are assumed to give out sounds that are precisely the same: the phase constants ϕ for the two waves are exactly the same. This is the tacit assumption of this example, due to, e.g., the two speakers hooked up to the same sound source. Such sources of waves, like these two loud speakers, are called **coherence sources**.

Example T16-15

In this example, the Doppler shift for the echo of a sound that bounces off a moving target is measured by the very person, who sent out the original sound. Please follow the solution given during the lecture, or in the book. Note that this problem involves two legs of motion: the first leg from the time that the sound goes off and arrives at the surface of the moving object, and the second leg from the time that the reflected sound goes off from the surface to the original person. In the first leg of the motion of the sound wave, this person is the sound source, while in the second leg of the motion of the sound wave, this person is the observer. In the first leg of the motion, the surface, at which the sound wave is reflected is the observer, while in the second leg of the motion, the surface is the sound source.

This example reminds me of a speed gun that a high way patrol uses to catch speeding cars! Indeed, the Doppler effect is what such a speed gun is based on; however, it is the Doppler effect of light (radio wave), not sound. **The Doppler effect of light is given by a completely different formula!** (And, it is given in the textbook, if you like to see it.)

Law of reflection

9.2 Law of reflection, law of refraction

As we start Chapter T32 and first consider a plane mirror, we use the law of reflection $\theta_i = \theta_f$.

You are quite encouraged to prove this law of reflection using Fermat's principle, as discussed on line as an extra credit question. Other ways include the momentum conservation and the following.



Proving the law of reflection

Note that the law of reflection ($\theta_i = \theta_f$) can be proven by using the “time-reversal symmetry principle.” According to this principle, if a physical process is possible, then another physical process which corresponds to the version of the first physical process, but played in reverse, just as you would play a movie backwards, is also a possible physical process, *if* no damping/heat-involving mechanism is considered. This symmetry principle holds for most microscopic laws.

9.3 Index of refraction

Due to the interaction of light and matter (electrons, mostly, and also other particles such as protons), light slows down in matter. This is summarized by

$$n \geq 1 \quad \text{index of refraction} \quad (9.1)$$

$$v = \frac{c}{n} \quad \text{speed of light in a medium} \quad (9.2)$$

$$\lambda = \frac{\lambda_v}{n} \quad (9.3)$$

where in the last equation, λ is the wave length in medium, while λ_v is the wave length in vacuum. As usual, here we are concerned with v of a travelling wave, not a standing wave ($v = 0$). This last equation above ensures that the frequency of a given

monochromatic light wave never changes (LN 6). Inside the medium, the frequency must be given by $f = v/\lambda$, and this matches the frequency in vacuum c/λ_v . Lastly, c is the speed of light in vacuum = 3.00×10^8 m/s, one of the most important constants of Nature.

Note that in some special frequency regime, some artificial materials give a negative index of refraction $n < 0$. They are interesting research materials, not considered in this course.

9.4 Snell's law

This law can be derived in a variety of ways, using Fermat's principle (as done in the forum), momentum conservation (as you can easily do *if* you know a bit of modern physics and make use of the above definition of the index of refraction; I am not requiring you to do this, yet, naturally), the Huygens/Feynman view of how light propagates (each point of a wave front is itself a new source of wave, and all of those waves interfere!), or simply by using the boundary condition that D must be continuous at the medium boundary (cf., LN 6). However, in this course, it is more important to know how to *use* Snell's law, than to know how to derive it. The law is given by

$$k_1 \sin \theta_1 = k_2 \sin \theta_2, \tag{9.4}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2. \tag{9.5}$$

See Figure T32-21 for the definition of symbols.

The first form, in terms of wave vectors, is valid for any wave, while the second form is specialized for light.