Notes for Lecture 8

Beats

8.1 Beats

Beats occur when two sounds with similar frequencies are superposed. Let us take

$$D_1(x,t) = A\sin(k_1x - \omega_1 t + \phi_1) \tag{8.1}$$
$$D_2(x,t) = A\sin(k_2x - \omega_2 t + \phi_2) \tag{8.2}$$

Using $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$, the superposed wave becomes

$$D = D_1 + D_2 = 2A\sin(k_ax - \omega_a t + \phi_a)\cos(k_bx - \omega_b t + \phi_b), \tag{8.3}$$
$$X_a = \frac{X_1 + X_2}{2}, \qquad X = k, \omega, \phi \tag{8.4}$$
$$X_b = \frac{X_1 - X_2}{2}. \qquad X = k, \omega, \phi \tag{8.5}$$

Assuming that $\omega_1 \approx \omega_2$, we see that $\omega_a \approx \omega_1 \approx \omega_2$ and $k_a \approx k_1 \approx k_2$ (since $k_1 = \omega_1/v, k_2 = \omega_2/v$). So, the sine function that occurs in the expression for D is just like one of the two original waves, up to a phase shift, to a good approximation.

Therefore, the superposed wave can be written as

$$D(x,t) = A_B(t)\,\sin(k_ax - \omega_a t + \phi_a), \tag{8.6}$$
$$A_B(t) = 2A\cos(k_bx - \omega_b t + \phi_b). \tag{8.7}$$

The key observation is that $A_B(t)$ is a slowly varying function than $\sin(k_ax - \omega_a t + \phi_a)$. Due to this, at any instant, one might hear the original frequency of the sound, but slowly the intensity of the sound that one hears will go up and down, following the

time dependence of $A_B(t)^2$ (recall from LN 6 that the intensity is proportional to A^2). This is the so-called **beat frequency**. $|A_B(t)|$ is often referred to as the envelope function—the wave $D(x,t)$ can be understood as the fast wave that is enclosed in the envelope function, which is slowly varying wave (cf., Fig. T16.30(b)).

Note that $A_B(t)$ has frequency ω_b. However, A_B^2 (or $|A_B|$) has half the period, and so it has double the frequency—this is the beat frequency by definition, since the beat frequency is the frequency that one hears, i.e., the frequency of the intensity.

$$\omega_B \;=\; 2|\omega_b| \;=\; |\omega_1 - \omega_2|. \qquad \textbf{beat frequency} \tag{8.8}$$