Notes for Lecture 7

Sound waves

7.1 Pressure wave

Sound wave is not only a displacement like a string wave, it is also a pressure wave. This is somewhat fascinating thing, actually, if one looks at the phenomenon of pressure microscopically. We will have a chance to give some serious consideration to this later in this course.

For now, let us just remember that

$$\Delta P = -B\frac{\partial D}{\partial x}. \tag{5.10}$$

Here, $\Delta P = P - P_{eq}$, and P_{eq} is the equilibrium pressure of the medium. Now, let us note that D satisfies the wave equation, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, derived in LN 5. By applying $-B\frac{\partial}{\partial x}$, from the left, to both sides of this wave equation and taking note of the fact that differential operations commute with one another we see that the same wave equation is satisfied by ΔP.

$$\frac{\partial^2 \Delta P}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Delta P}{\partial t^2}. \tag{7.1}$$

So, the sound wave can be described as pressure wave as well as displacement wave.

If the amplitude of a sinusoidal wave D is A, then the amplitude of ΔP is given by

$$\Delta P_{max} = BkA. \qquad \text{amplitude for } \Delta P \tag{7.2}$$

This is because if, e.g., $D = A\sin(kx - \omega t + \phi)$, then

$$\begin{aligned} \Delta P &= -BAk\cos(kx - \omega t + \phi) \quad (7.3) \\ &= BAk\sin\left(kx - \omega t + \phi - \frac{\pi}{2}\right). \quad (7.4) \end{aligned}$$

The last expression also shows that the shape of ΔP wave is phase shifted by $-\pi/2$ relative to the shape of D, for a plane wave.

7.2 Standing waves in sound instruments

Only discrete values of wave lengths are allowed for musical instruments due to the boundary condition (read the last section of LN 6 to learn the origin of these boundary conditions come from). For a guitar string of length l, the boundary condition is that D must have nodes ($D = 0$). This means that λ values are restricted to:

$$n\lambda_n = 2l \qquad n = 1, 2, 3, \ldots \quad (7.5)$$

is the condition for the standing waves.

The same equation holds for the sound wave in an open tube. Or, a closed tube. For sound waves in a half-open tube, with one end closed and the other end open, of length l, one can derive that

$$(2n-1)\lambda_n = 4l \qquad n = 1, 2, 3, \ldots \quad (7.6)$$

is the condition for the standing waves.

The boundary conditions for an open tube are that $\frac{\partial D}{\partial x} = 0$ at both ends. The boundary condition for a closed tube end is that $D = 0$ at that end. The above conditions for discrete values of λ_n can be derived from these boundary conditions (suggestion: put the open end at $x = 0$ and then write down the standing wave form as $D_s(x) = A(t)\sin(kx)$, then apply the boundary condition for $D_s(l)$ or $\frac{\partial D_s}{\partial x}(l)$). Note that the frequencies corresponding to them, $f_n = v/\lambda_n$ are **natural (resonant) frequencies** of the tube/string. Note that $v = 0$ for a standing wave, but in the expression, $f_n = v/\lambda_n$, v is the speed of a single travelling sinusoidal wave (so *not* zero!).

Why these boundary conditions? It is possible to reason like in the last section of LN 6. For sound wave, note that an open end is a free end for D. So, it is an **anti-node** ($|D| = \max$) for D, which means a **node** for ΔP ($\Delta P = 0$) due to Eq. 5.10. For a closed end, it is a fixed end for D: i.e., it is a node for D ($D = 0$) and an anti-node for ΔP ($|\Delta P| = \max$). From the pressure point of view, the closed end is a free end, since the system can take any pressure there. On the other hand, at an open end, the pressure must negotiate with the air outside and comes to the equilibrium value, or very close to it, within a very short distance, giving a node for ΔP.

Potentially very misleading diagrams

I don't know about you, but when I look at the diagrams shown in pages T434 and T435, an idea gets into my head. My brain recognizes that thick blue lines as pipes (musical instruments), and that red lines represent the wave amplitude D for the sound wave. If literally interpreted, then these diagrams suggest that D is on the order of the size of the pipe!

This is very far from true!

Example T16-7, which we went through during the lecture, shows that the displacement and the pressure variation are truly minute! It is mind-boggling how we can perceive such small variations (and how about other animals, which are better than us!). Even for really really loud sound (cf. Table T16-2) that may break the instrument, D_{max} is only about 10^{-4} m = 0.1 mm. So, one must keep in mind that D is *really small* compared to the size of the pipe, and in the longitudinal direction. To give the benefit of doubt, we must consider diagrams shown in Figures T16-11 and T16-12 as only *very crudely symbolic*, as far as the direction of D and the magnitude of D are concerned.