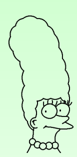
Notes for Lecture 1

Hooke's Law and SHM

1.1 The notes

My lecture notes will be short and sweet. At least I will try to make them to be so. These lecture notes are meant to be a short summary of what I discussed in class. However, all essential mathematical and physical steps will be included.

Read my lecture notes after class, while you do homework, and before preparing for exams, to clear up things that may be still confusing to you. Before each class, you must read the textbook, and do the reading quiz.



The numbering convention for referred contents

Sometimes figures, equations, sections, etc. of the textbook may be referenced here. In such a case, the figure/equation/... number will be preceded by the letter T. So, if you see "Figure T14-3," that means a figure with the serial number 14-3 in the text book—so you need to open your book! "Figure 14.3" would be a figure 14.3 of these lecture notes.

In my Lecture notes, "Simpson boxes" will be used with the following convention.

Marge boxes contain essential things to know. Homer boxes contain essential warnings against pitfalls or common mistakes. Lisa boxes contain optional advanced contents. Bart boxes contain maths or tricks, and their contents are also considered advanced and optional. Often, Lisa boxes and Bart boxes contain contents that are necessary, but forgettable[1], ingredients for rigorously proving what we do.

1.2 What we will cover in this course

I think one way to characterize this course is that it is a course on "*classical* wave mechanics." For the most part, we will be concerned with the wave phenomena. And here, we deal with wave phenomena that human beings can experience by sight, hearing or touch. This is the meaning of "classical"[2].

This is as opposed to "quantum mechanics," which is often called, simply, "wave mechanics." Since this is the first lecture, let me say a few words about quantum mechanics. However, from Lecture 2, I will never mention quantum mechanics, to help us focus on what we have in front of us, not what is ahead of us. Now, quantum mechanics is, at its heart, the science of invisibly, inaudibly, and untouchably small things, like an electron or an atom. That small world is such an alien world from our ordinary world. It is a world that we human beings find it very hard to "understand" or "relate to." There is one thing that we know for sure, though: *everything* acts like wave, in that (really) small world! This is why quantum mechanics is simply called wave mechanics. Also, this is the reason why classical *wave* mechanics that we will cover in this course is so very helpful as a stepping stone towards quantum mechanics. A lot of core concepts that we learn in this course are applicable in quantum mechanics: they include wave propagation, group velocity, interference, diffraction, polarization, etc. Quantum mechanics becomes easier to digest if one starts from the classical wave mechanics point of view.

Of course, classical wave mechanics is a very fascinating science, by itself, so we don't necessarily need quantum mechanics to motivate us here. However, it could not hurt to already have some feeling about quantum mechanics.

[1] For the purpose of this course, that is. Not in the grand scheme of things!

[2] Here, I am speaking in laymen terms. In physics terms, "classical" means either Newton's classical mechanics or Maxwell's classical electrodynamics.

1.2.1 Hooke's law

Before we discuss any wave, we must know the simple harmonic oscillation (SHO). The origin of the SHO is the Hooke's law force.

Consider a spring with a mass m, attached to the end of it. We consider the spring to be horizontal. You pull it a bit, say by $x > 0$, and stop and hold. At this moment, the mass is not moving at all. So, the net force on mass m must be zero. The net force = $F_h + F_s$, where F_h is the force by hand on mass m, and F_s is the force by spring on mass m.

$$F_s + F_h = 0 \qquad \text{horizontal spring, h = hand, s = spring} \tag{1.1}$$

Now, consider a vertical spring with a natural length l_0 (no load). Attach mass m to the lower end of it, and we see that $l_0 \to l_0 + x_0$ ($x_0 > 0$). By a similar reasoning as above, we must have

$$F_s + mg = 0 \qquad mg \text{ is the force of gravity on mass } m \tag{1.2}$$

Experimentally, we find that $F_s = -kx_0$ if x_0 is small enough. So, we have

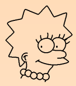
$$kx_0 = mg \qquad \text{vertical spring with mass } m\text{, in equilibrium} \tag{1.3}$$

where x_0 is the length increase of the spring when m is attached to the lower end of the spring.

For a spring that is compressed or extended by x relative to its equilibrium distance, the spring force is given by Hook's law

$$F_s = -kx \tag{1.4}$$

k is a **spring constant** (unit = N/m). It is customary to define the sign of x such that $x > 0$ means extension, and $x < 0$ means compression. In any case, the negative sign in the above equation means that the force, or the acceleration, points opposite to x.



Hooke's law is an approximation

That Hooke's law is just an approximation can be seen, e.g., by imagining a huge load on a tiny spring. The spring will get extended and will be permanently deformed. As we will see in the next section, such deformation is *not* consistent with Hooke's law, according to which a simple harmonic motion must result. A motion involving a permanent deformation is most definitely *not* a simple harmonic motion.

Related to this, note that in the above discussion, Eqs. 1.1,1.2 are more general than Eq. 1.4. What I mean is the following. A spring satisfying Eq. 1.4 necessarily satisfies Eqs. 1.1,1.2 in the respective equilibrium setups. However, *any* spring must satisfy Eqs. 1.1,1.2 in the respective equilibrium setups, even if the spring disobeys Hooke's law.

If Hooke's law is just an approximation, how come it is valued as a *law*? Good question. It is because for *every stable mechanical system*, the Hooke's law force results if the system is displaced from the equilibrium by a small amount. This universality is the reason for calling it a *law*!

1.3 Hooke's law and SHO

The motion that results from a Hooke's law force, and only that force, is called the simple harmonic oscillation (SHO).

Not any oscillation is a SHO.

Just because there is an oscillation does not mean it is a SHO! Only an oscillation caused by the Hooke's law force alone is a SHO.

What this means is that the solution to Newton's second law equation of motion (EOM)

$$m\ddot{x} = -kx \qquad \text{SHO, EOM} \qquad (1.5)$$

defines the SHO, or the simple harmonic motion (SHM).

The **general solution** to the above EOM is given by any one of the following three forms[3].

$$x(t) = A\cos(\omega t + \phi) \qquad A \geq 0 \qquad (1.6)$$
$$x(t) = B\sin(\omega t + \phi') \qquad B \geq 0 \qquad (1.7)$$
$$x(t) = C\cos(\omega t) + D\sin(\omega t) \qquad (1.8)$$

where A, B, C, D, ϕ, ϕ' are constants, independent of k and m, and

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{angular frequency} \qquad (1.9)$$

The proof of the equivalence of the above forms is left for your exercise. The following information will be quite helpful for your proof.

Trigonometry

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b \qquad (1.10)$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \qquad (1.11)$$

That each of Eqs. 1.6,1.7,1.8 is a solution for the EOM, Eq. 1.5, can be verified by direct substitution. This verification is also left for your exercise.

[3]These three forms do not exhaust all possible forms.

Two for second derivative

The fact that there are two constants (A, ϕ or B, ϕ or C, D, respectively) for each of the above general solutions comes from the fact that the EOM, Eq. 1.5, is a second order differential equation, i.e. the highest derivative is the second order derivative. This fact remains true for any force, not just for Hooke's law force. The proof of this fact is beyond the scope of this course, although I sketched it in class. You will learn the proof in an advanced course. For example, see pages 6,7,8 of this pdf file, for a preview.

Eq. 1.6 is a quite common form to use, while not the only form to use. For pedagogy, we will benefit from sticking with it, and so we will do so from now on. Eqs. 1.7,1.8 will be rarely used, if ever.

Those are Greek!

ω is omega, not double-u. ν is nu, not v.

1.4 SHO

Let us summarize the essence of a SHO.

$$\ddot{x} = -\omega^2 x \qquad \text{SHO, EOM} \tag{1.12}$$

$$x = A\cos(\omega t + \phi) \qquad \text{SHO, general solution} \tag{1.6}$$

Here, the EOM is written in a simper way, using Eq. 1.9. The new form is more fundamental.

ω is also referred to as **natural (angular) frequency**. Note the fundamental nature of ω, since that is the only parameter that enters the SHO EOM, as written here in a simpler and more fundamental way than Eq. 1.5.