Due Feb. 27, Thursday

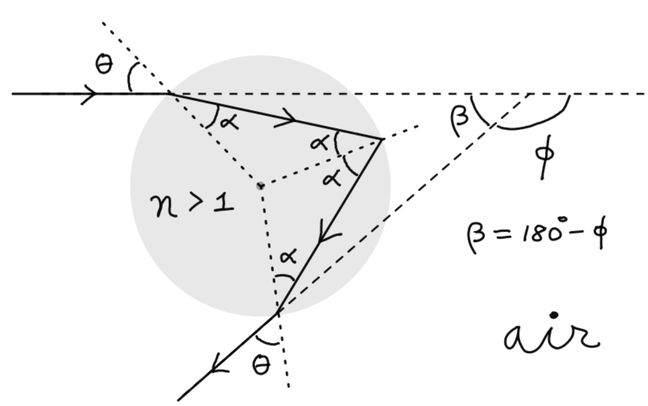
All problems must be solved symbolically first. Then, any numerical answer, when required, can be computed by substituting numbers into the symbolic expression at/near the very end. Solving problems symbolically means deriving the answer in terms of symbols, instead of numerical values. All problem numbers refer to those in the textbook. (Not all problems may be graded in detail, due to limited man power; however, you must do all problems.)

For each problem, you are required to use sensible symbols, by defining or adopting them yourself, for your symbolic solution. If you are unsure how to do so, feel free to ask (or look back at homework 1)!

Problem 1 (10 points) Problem 32.54 (Snell's law; prism; dispersion).

Problem 2 (10 points) Problem 32.55 (Snell's law and water drop; dispersion).

Problem 3 (30 points) [Rainbow] Consider a sphere made of material with index of refraction $n > 1$ (e.g., a water drop hanging in air). A ray of light hits the upper half of the sphere, as shown in the diagram below. Of all possible paths that this ray of light can follow subsequently, we focus our attention on the particular kind of path that is depicted below: the events that define such path is a refraction, an internal reflection, and then a refraction. The "scattering angle" ϕ, which measures the degree of deflection of the beam by the chain of these events, or its complementary angle β, is of our utmost concern.



(a) Using Snell's law and the law of reflection, (i) find the function $\phi = \phi(\theta, n)$, and (ii) verify that $\phi(\theta = 0, n) = 180°$ and $\phi(\theta = 90°, n = 1.33) = 165°$ (as shown for these two special angles in class).

(b) Show that $f(\theta) = \phi(\theta, n)$ has a minimum as a function of θ, at

$$\theta_m = \sin^{-1}\sqrt{\frac{4-n^2}{3}}, \quad 0 < \theta_m < 90^\circ,$$

for the given range of $\theta:\ 0 \le \theta \le 90^\circ$, if $n < 2$. [Hint: $\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}}$.]

(c) That the function $f(\theta) = \phi(\theta, n)$ has an extremum at θ_m is an important fact, for the following reason. It is possible to show that the intensity of light that appears between angles ϕ and $\phi + \Delta\phi$ (for small $\Delta\phi$) is given by $G(\theta)\Delta\theta \approx G(\theta)\Delta\phi/\left|\frac{df}{d\theta}\right|$, where $G(\theta)$ is a regular (i.e., non-divergent) function of θ. Note that $\left|\frac{df}{d\theta}\right|$ vanishes at an extremum point of $f(\theta)$, giving rise to a *divergent* intensity at $\theta = \theta_m$. This means a singular intensity maximum at $\theta = \theta_m$ or at the corresponding $\phi_m = \phi(\theta = \theta_m)$. So, this is the reason why the extremum angle ϕ_m, or $\beta_m = 180^\circ - \phi_m$, is singularly important. Now, we shall assume water droplets only. Under ambient conditions, the refractive index (n) of water varies monotonically from 1.330 (red) to 1.343 (violet) for the visible spectrum of light. Calculate the values of $\beta_m \equiv 180^\circ - \phi_m$ for red light and violet light, and thus show that the values of β_m for the visible spectrum of light span about 2 degrees, with maximum β_m for red and minimum β_m for violet—this explains the main arc of a rainbow.

Problem 4 (10 points) Problem 32.74 (Snell's law).

Problem 5 (10 points) Problem 32.83 (Total internal reflection).

Problem 6 (10 points) Problem 33.10 (Lens).

Problem 7 (10 points) Problem 33.20 (Lens, combination).

Problem 8 (10 points) Problem 33.98 (Glasses).