

Barebone review for the final (part 2; midterm review is part 1)

Law of reflection: $\theta_r = \theta_i$ (specular reflection), for a smooth surface. Origin of all mirror equations. If $\theta_i = 0$ (or not too large), and if the second medium that light is bouncing off of has a greater index of refraction, then there will be a π phase shift.

Index of refraction: $n \geq 1$. $v = c/n$. $\lambda_m = \lambda_v/n$. λ_m is the wave length inside the medium with refractive index n . λ_v is the wavelength in vacuum, and is often used to name light, as in “550 nm light,” even when light travels in $n > 1$ medium. If n depends on wave length, then we get **dispersion** (rainbow, prism, optical communication).

Law of refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ (Snell’s law)—the basis of total internal reflection (if $\theta_1 \geq \theta_c = \sin^{-1} \frac{n_2}{n_1}$), apparent height (reduced by $\frac{1}{n}$ if observed from air), dispersed light from prism (red bends the least), rainbow scattering, and all lens equations.

The frequency of light (or any wave) does not change upon reflection, or (re/dif)fraction.

Spherical/plane mirrors $f = r/2$, where r is the radius of curvature. Sign convention: $r < 0$ for convex mirror, and $r > 0$ for concave mirror.

Lensmaker’s equation for thin spherical/plane lens: $\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$, where R_1 and R_2 are the radii of curvature for the two sides. Sign convention: $R_i > 0$ for convex (convergent) surface, and < 0 for concave (divergent) surface. f is left-right symmetric.

Diopter: The SI unit (D) for the power of lens: $P \equiv f^{-1}$ (including sign!).

The master equation for mirror or lens: $f^{-1} = d_o^{-1} + d_i^{-1}$. Sign conventions: d and f are positive if on the “right side” (= real) and negative if on the “wrong side” (= virtual). Or, $d_o > 0$ (< 0) if the input beam diverges (converges) on the optical element, and $d_i, f > 0$ (< 0) if the image beam is convergent (divergent) from the element. The zero point for d and f values is the position of the optical element.

Multiple optical elements: Treat one at a time. The image of one element becomes the object of the next element. Real or virtual character can change when applying this “chain rule.” If thin elements are stacked to form a compound thin lens/mirror: $f_{compound}^{-1} = \sum_i f_i^{-1}$.

Lateral magnification: $m \equiv h_i/h_o$. For a *single* optical element: $m = -d_i/d_o$, a real image is inverted, and a virtual image is upright.

Angular magnification, magnifying power: $M \equiv \theta'/\theta$, where θ' is the angle at which light enters the eye, fitted with the optical device, and θ is the angle at which light enters the bare eye with no optical device (with object at $N = 25$ cm, the normal near point, when possible). $M = N/f$ for a simple magnifying lens with relaxed eye. $M = 1 + N/f$ for maximally strained normal eye. For a simple refracting telescope, $M = -f_o/f_e$, where f_o is the focal length of the objective, and f_e is the focal length of the eyepiece.

Thin film interference: Not only the path length difference ($2t$ with $t =$ film thickness), but also any π phase shifts for the two reflections involved must be considered. $2t = m\lambda_f$ (λ_f is the wavelength *inside* the film) gives the constructive condition if π phase shifts occur for both or none of the two reflections, but it gives the destructive condition if π phase shift occurs only for one of the two reflections. $2t = (m + 0.5)\lambda_f$ gives the other condition.

Single slit (rectangular): $D \sin \theta = m\lambda$ (destructive), $m = \pm 1, \pm 2, \dots$. **This equation with $m = 0$ is a constructive interference condition, giving by far the brightest fringe.** Other constructive interferences: $D \sin \theta \approx \pm \left(m + \frac{1}{2}\right)\lambda$ with $m = 1, 2, 3, \dots$. Thus, not only the central bright fringe the most intense, it is also twice wider. All of these derive from the intensity: $I = I_0 \frac{\sin^2(\beta/2)}{(\beta/2)^2}$, where $\beta = kD \sin \theta$ and $k = 2\pi/\lambda$. β is the phase difference

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for the two beams of light emanating from the two end points of the slit. $D \sin \theta = m\lambda$ is equivalent to $\beta = 2m\pi$, and $D \sin \theta \approx \pm \left(m + \frac{1}{2}\right)\lambda$ to $\beta \approx \pm (2m + 1)\pi$.

Single slit (circular, diameter D): Airy pattern. The first minimum occurs at $D \sin \theta = 1.22\lambda$, instead of $D \sin \theta = \lambda$ (rectangular slit; see above).

Double slit (very narrow slits): $d \sin \theta = m\lambda$ (constructive). $d \sin \theta = \left(m + \frac{1}{2}\right)\lambda$ (destructive). $m = \text{any integer}$. With $\delta \equiv kd \sin \theta$ (the phase difference), these conditions read $\delta = 2m\pi$ and $\delta = (2m + 1)\pi$, respectively. All of these derive from $I = I_0 \cos^2(\delta/2)$.

Double slit (rectangular, finite width D): $I = I_0 \cos^2\left(\frac{\delta}{2}\right) \frac{\sin^2(\beta/2)}{(\beta/2)^2}$. The (slowly varying) single slit pattern, modulated by the (fast changing) double slit pattern.

Diffraction grating or N slits: $d \equiv \text{groove/slit spacing}$. Principal maxima given by $d \sin \theta = m\lambda$ ($m = 0, \pm 1, \pm 2, \dots$). Minima given by $d \sin \theta = \left(m + \frac{n}{N}\right)\lambda$ where $n = 1, \dots, N - 1$ for *each* value of m . So, between two adjacent principal maxima, $\exists N - 1$ minima. Approximately midway between any two adjacent pair of these minima, \exists a low intensity local maximum. All of these derive from one equation: $I = I_R \cdot \frac{\sin^2 \frac{N\delta}{2}}{\sin^2 \frac{\delta}{2}}$, where $\delta \equiv kd \sin \theta$, and $I_0 = I_R N^2$ is the peak intensity for *any* principal maximum. I_R is independent of N . I_0 scales as N^2 . Peak width scales as $1/N$.

X-ray diffraction: Nature's gift of infinite diffraction gratings is a (small) crystal. $2d \sin \phi = m\lambda$ ($m = 0, \pm 1, \pm 2, \dots$) gives **Bragg diffraction** condition, where d is the spacing between lattice planes. ϕ is measured relative to the lattice plane, *not* its normal.

An overarching only assumption for slit/diffraction setup: l (the slit/grating to screen distance) is much greater than any slit/grating dimension (D, d, Nd , crystal size).

Resolution: For a circular single slit, the angular resolution due to diffraction is given by $\theta = 1.22\lambda/D$ (Rayleigh criterion; assuming small θ). This applies to any circle-shaped optical element. Or, qualitatively speaking ($\theta \sim \lambda/D$), to any thing size D that bends/reflects/emits light. For lateral resolution, multiply θ by the relevant distance. Diffraction grating has resolution $\Delta\lambda = \lambda/(N|m|)$ for $m = \pm 1, \pm 2, \dots$. The **resolving power** $\equiv \lambda/\Delta\lambda = N|m|$.

Linearly polarized light: can be generated and detected by a conducting rod (or an array of such rods/long-molecules, as in a Polaroid sheet). The electric field, \vec{E} , oscillates along one particular axis. If a linearly polarized light passes through an ideal linear polarizer, $I = I_0 \cos^2 \theta$, where θ is the angle between the incoming light polarization $\vec{\epsilon}_{li}$ and the polarization axis of the polarizer ($\vec{\epsilon}_p \perp \text{rods}$). The outgoing light is polarized along $\vec{\epsilon}_{lf} = \vec{\epsilon}_p$.

Brewster angle: $\theta_B + \theta_{refraction} = 90^\circ$: the reflected light is linearly polarized 100 % parallel to the surface/interface. $\tan \theta_B = \frac{n_2}{n_1}$, for light going from medium 1 to medium 2.

Fluid: Buoyant force = the weight of the displaced fluid (Archimedes). Pressure $P = F/A$: force (F) is normal to any surface (area A) present. $dP = -\rho g dy$ (fluid in a gravitational field; compressible or not). $P = P(y=0) - \rho g y$, if incompressible. If the fluid is moving, $P + \frac{1}{2}\rho v^2 + \rho g y$ is conserved for a laminar non-viscous flow of an incompressible fluid (Bernoulli; energy conservation; here, P is the pressure measured in the moving fluid frame).

Statics: Equilibrium requires zero net force and zero net torque. $\vec{F} = m\vec{a}$ is your friend, especially $m\vec{a}$ (start your reasoning from it!). The center of gravity (CG) is *not* a well-defined concept, in general; it arises from a *desire* to describe the net torque due to gravity in a simple manner. In uniform gravity, CG = center of mass.