# Simple Harmonic Motion (SHM)

$$\ddot{x} = -\omega^2 x, \qquad x = A\cos(\omega t + \phi), \qquad \omega = 2\pi f = 2\pi/T$$

- $\omega = \sqrt{k/m}$ (mass on spring, horizontal or vertical; $F_{net} = -kx$; the gravity effectively disappears from the problem in the vertical spring problem if $x$ measures the displacement relative to the new equilibrium *with* mass on spring).
- $\omega = \sqrt{\kappa/I}$ (rotation; pendulum or torsion oscillator; torque $\tau_{net} = -\kappa\theta = I\ddot{\theta}$, $|\theta| \ll 1$).
- $\omega = \sqrt{mgl/I}$ (any physical pendulum; $\kappa = mgl$, $l$ = distance from pivot to the center of mass), $\omega = \sqrt{g/l}$ (simple pendulum; $I = ml^2$).
- For pendulum or torsion oscillator, $\theta = \theta_{max}\cos(\omega t + \phi)$, and $\omega \neq \dot{\theta}$.

**Other kinematic quantities**

$$\dot{x} = -\omega A\sin(\omega t + \phi) \qquad \text{velocity; max speed} = A\omega$$
$$\ddot{x} = -\omega^2 x = -A\omega^2\cos(\omega t + \phi) \qquad \text{acceleration; max acceleration} = A\omega^2$$

**Energy conservation**

$$E = K(t) + U(t) = U_{max} = K_{max} = \frac{1}{2}m\omega^2 A^2 \text{ or } \frac{1}{2}I\omega^2\theta_{max}^2 = \text{constant in time}$$
$$U(t) = \frac{1}{2}m\omega^2 x^2 \text{ or } \frac{1}{2}I\omega^2\theta^2, \qquad K(t) = \frac{1}{2}m\dot{x}^2 \text{ or } \frac{1}{2}I\dot{\theta}^2$$

# Wave

Any wave can be broken down into travelling sinusoidal waves (Fourier series/integral). A **travelling sinusoidal wave** (a "**plane wave**") and a standing sinusoidal wave are of our utmost concern. A **spherical wave** or a circular wave is also of our concern, and is a more realistic wave from a small source in three or two dimensions. However, locally, they can be approximated as plane waves.

$$D_\pm(x,t) = A\sin(kx \pm \omega t + \phi) \qquad \text{travelling sinusoidal wave}$$
$$D_s(x,t) = D_+ + D_- = 2A\sin(kx + \phi)\cos(\omega t) \qquad \text{standing sinusoidal wave}$$

The travelling sinusoidal wave has a sine shape and it is moving at the **wave speed** $v = \omega/k$ (this formula valid **only for** travelling sinusoidal wave) right ($-$) or left ($+$). The standing sinusoidal wave has a sine shape which is not moving, but experiencing a "time dependent ~~amplitude/~~ scaling-factor" $2A\cos(\omega t)$: so it is "breathing," rather than moving. So, the wave speed for a standing wave is zero. In either case,

$$\lambda = \frac{2\pi}{k}. \qquad \text{wave length } (\lambda)\text{, wave number } (k;\ \textit{not } k \text{ in Hooke's law force } -kx!)$$

A **transverse wave** (e.g., a string wave) is a wave for which $D$ and the wave velocity (or $k$ = wave number, or wave *vector*) are perpendicular to each other, and a **longitudinal wave** (e.g., a sound wave in air/water) is a wave for which $D$ and the wave velocity (or $k$ = wave number, or wave *vector*) are along the same axis.

For the above sinusoidal waves (travelling or standing), the local motion, for fixed value of $x$, is just a SHM (just examine $D_\pm$ and $D_s$ as a function of $t$)! Physically, this means that a sinusoidal wave is a coordinated SHM, where each *particle* defined by the small segment between $x$ and $x+dx$ is going through a SHM and this motion transpires to its neighbors. For this reason, $\omega$ (or $f$) **is a preserved quantity** when a wave propagates/reflects/refracts/diffracts/etc. in an inhomogeneous medium environment. The **particle velocity** is given by $\partial D/\partial t$, and should not be confused with the **wave velocity**. For travelling wave (sinusoidal or not), the **wave speed** is given by $v = \sqrt{F_T/\mu}$ (string wave), or $\sqrt{B/\rho}$ (sound wave in gas/liquid), etc.

**Superposition principle** is the fundamental principle of waves. It is best viewed as a mathematical principle. **Interference** can be thought of as a physical realization of the superposition principle: when two waves ($D_1$ and $D_2$) meet, then

$$D = D_1 + D_2. \qquad \text{fundamental for any coexisting waves}$$

**Energy in a wave**: $E = \frac{1}{2}\rho S L \omega^2 A^2 = 2\pi^2 \rho S L f^2 A^2$ (for a travelling sinusoidal wave over a length $L$; medium cross section surface area $S$; the SHM energy since $\rho S L$ is the total mass). $P = E/T$ ($T$ is the time of travel for $L$), and $I = P/S = \frac{1}{2}\rho\omega^2 A^2 v$. These are only for the plane wave. A point source in three dimensions will result in a spherical wave: $I \propto r^{-2}$ since $S = 4\pi r^2$ and $P$ = preserved.

**Reflection, boundary condition, and phase shift:** If $D$ is constrained to be zero (fixed end), then the wave reflects while changing sign ($\pi$ phase shift). If $D$ has no constraint (free end), then the reflected wave involves no sign change (no phase shift).

# Sound

**Sound wave:** Displacement wave, as well as pressure wave. Departure of the pressure from the equilibrium pressure: $\Delta P = -B\,\partial D/\partial x$. All other general properties of the wave holds for sound wave ($v = \lambda f$, interference, etc).

**Beats:** Arise from $\sin(\omega_1 t + \phi_1) + \sin(\omega_2 t + \phi_2) = 2\sin(\omega_a t + \phi_a)\cos(\omega_d t + \phi_d)$, where $\omega_a = (\omega_1 + \omega_2)/2$, $\omega_d = (\omega_1 - \omega_2)/2$, $\phi_a = (\phi_1 + \phi_2)/2$, $\phi_d = (\phi_1 - \phi_2)/2$. If $\omega_1 \approx \omega_2$, then this superposed wave has an "envelope" function, $\cos(\omega_d t + \phi_d)$, which varies slowly in time. This causes the sound *intensity* oscillate slowly in time with angular frequency $= 2|\omega_d| = |\omega_1 - \omega_2|$. This is **the beat frequency**: $\omega_B = |\omega_1 - \omega_2|$, or $f_B = |f_1 - f_2|$.

**Decibel:** Logarithmic intensity scale (dB), $\beta \equiv 10\log_{10}\frac{I}{I_0}$, where $I_0 = 10^{-12}$ W/m$^2$.

**Standing waves:** For a guitar string or an open (or closed) tube, of length $l$, we get $n\lambda_n = 2l$. For a half open tube, $(2n-1)\lambda_n = 4l$. Here, $n = 1, 2, 3, \ldots$. At an open end of tube, pressure $\Delta P$ ($D$) has a node (anti-node). At a closed end of tube, displacement $D$ ($\Delta P$) has a node (anti-node). The frequencies that correspond to these discrete wave lengths, $f_n = v/\lambda_n$, are examples of **resonant frequencies** or **natural frequencies**, at which the object responds with singular sensitivity.

**Doppler effect:** $f' = \frac{v \pm v_o}{v \pm v_s} f$, where $v_o$ ($v_s$) is the speed of the observer (source) of sound, and $v$ is the speed of sound. So, $v, v_o, v_s$ are all positive, and the signs ($\pm$) in front of $v_o$ and $v_s$ must be chosen based on physics. The sound medium must be at rest.