Notes for Lecture 20

Diffraction

The thing about diffraction is that it occurs everywhere. Looking up dictionaries, I learn that the word diffraction drives from a Latin word meaning "shattering, breaking apart."

How do we "break apart" a beam of light? Oddly enough, what we need to do is very simple – just make the beam small by using a mask! In a single slit diffraction experiment, the diffraction occurs since the slit plate masks the majority of light, and the beam profile right after the slit plate looks just as we "ordered" – the crossection of the beam looks just like the slit that we made light go through. Are there any other thing that we do to light than simply masking the beam when we place a slit plate on a beam of light? Sure, depending on what the slit material is we might be doing some additional thing such as causing some light to reflect off the inner edge of the slit. However, clearly such effect can be minimized if we use a weakly reflecting material, and we shall thus ignore such a side effect. So, the role of a slit plate is masking an incoming beam of light, so that its cross section is of a certain shape that we like.

To be quantitative, it helps to consider a simple rectangular slit of dimensions $D \times L$. We will often call D the width of the slit. L is the other side ("height") of the rectangle. We take the coordinate system such that D is along the y axis, and the slit plate and the screen are assumed to be parallel to each other (as in our double slit setup), and both are perpendicular to the x axis. So, in this setup, the parameter l (the distance between the slit plate and the screen) is an x coordinate value.

In this coordinate system, the center of the slit is considered as the origin, and the L side of the slit is along the z axis. We shall focus attention to the xy plane only ($z = 0$). We shall also assume that L is either very small or very large. In either case, we do not need to consider the effect of L to figure out the pattern at $z = 0$ (proving this statement is left for your exercise).

With this standard setup, we can prove, as we did in class, that

$$D \sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots \quad \text{(Destructive interference)} \tag{20.1}$$

$$D \sin\theta = 0 \quad (\text{i.e., } \theta = 0) \qquad \underline{\text{Maximum constructive}} \text{ interference} \tag{20.2}$$

$$D \sin\theta \approx \pm\left(m + \frac{1}{2}\right)\lambda \qquad m = 1, 2, \ldots \quad \text{(Other constructive interference)} \tag{20.3}$$

Note that the "other" constructive interferences, indicated by the last equation give quite weak peaks (cf. Example 35-3 of the book) compared to the central peak due to the maximum constructive interference condition. Note also that the central peak, which occurs between $m = \pm 1$ destructive interferences, is about twice larger than the other maximum peaks, which occur about half way between two destructive interference conditions corresponding to two adjacent m values with the *same* sign. LN 21 has a detailed mathematical expression (Eq. 21.36) from which the above formulas can be derived.

The result is that most intensity falls into the central peak which is twice wide as other peaks of intensity. Therefore, the first zero intensity condition

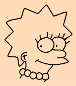
$$D \sin\theta = \lambda \tag{20.4}$$

is important, since it provides the size of the central peak.

The more you squeeze, the quicker it will expand.

Let us think about what the last equation means in terms of how much you "squeeze" the beam of light, and how much the beam diffracts, i.e. how large it becomes on the screen, responding to this squeezing. The smaller D is, the more we squeezed the beam. The larger the value of $|\theta|$ (with an upper limit of $\pi/2$, of course, enforced by the setup), the larger the diffraction spot, since $y = l \tan\theta$ will become larger in magnitude when $|\theta|$ increases. From the above equation, $D \sin\theta = \lambda$, it is clear that the more you squeeze the beam (small D), the more it diffracts (large θ). Lastly, note that we did not really "squeeze" the beam, since we simply masked out the majority of light and reduced the cross section of the beam to a small size. However, the physics of diffraction is independent of the incoming intensity of the beam, and so the same diffraction phenomenon will occur also, *if* we somehow squeezed a large cylindrical beam of light to a small cylindrical beam of light, using two lenses!

Note that we do *not* really need a slit to observe diffraction. As long as we can impart the same initial condition (as the beam's initial condition just after it comes out of the slit) *somehow* using other means than using a slit, then the same diffraction effect will occur! (cf. clicker quiz, 1st question.)



Heisenberg uncertainty principle

This famous principle is valid for all kinds of waves, not just for a quantum particle. On the other hand, what we call "light" (that is, classical light) is really just a lot of what we call "photons" (that is, quantum particle), and so the Heisenberg principle is expected to be valid no matter how you look at light (as classical wave, or quantum particle). The more you squeeze the beam, the more precise the position of light. It then follows from the uncertainty principle that the momentum of light becomes more uncertain, in other words its momentum values become more broadly distributed. In this example, we are talking about the y position and the momentum along the y direction, p_y, only. Note that, initially, right out of the slit, the beam does not really have any sense of movement along the y direction. However, the uncertainty principle requires that p_y, while averages to zero, will have a broad distribution of finite values. Importantly, the width of this distribution, Δp_y, is inversely proportional to $\Delta y \approx D$. A large value of Δp_y means a faster spreading in the y direction, while the mean position stays the same ($y = 0$). So, this can be understood as the reason why the beam expands faster and produces a bigger diffraction spot, when D is made smaller! Of course, this would be valid for all waves that we considered so far, e.g., the sound wave. So, the Heisenberg uncertainty principle is not just for quantum mechanics. At the same time, we do *not need* to know it to explain diffraction, as, clearly, we did not use the Heisenberg uncertainty principle to find all the things that we found above, and so if the mentioning of this principle makes your head ache, and makes you feel under pressure, I understand – you can simply forget about this principle, until you learn it in more depth in other courses. For this course, you will be just fine without knowing it.

For those of you who do happen to know the Heisenberg uncertainty principle well and, perhaps, a bit of modern physics as well, here are just a few more words. A finite Δp_y means a finite Δk_y (wave vector along y), and a finite $\Delta \lambda_y$ (wave length along the y direction). So, while the λ value along the x direction remain the same, light becomes a wave packet consisting of a distribution of λ_y values as far as the y degree of freedom is concerned, in the above example. So, *overall*, the beam is no longer monochromatic! One can easily generalize this argument: only a sinusoidal (plane, spherical, or else) wave that has an *infinite* extent in time and space is truly monochromatic. Indeed, this well-known fact in wave theory can be considered as an example of the Heisenberg uncertainty principle at work.