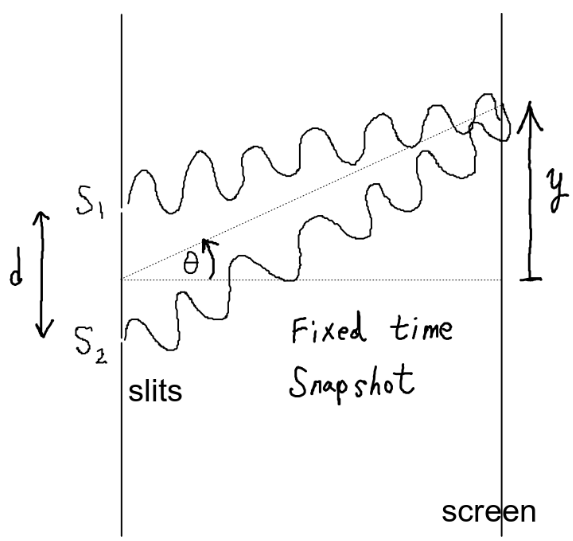
Notes for Lecture 16

Young's double slit experiment

Again, you must read the book and your lecture notes taken during the lecture to go over all basic materials. Here, let us summarize a more rigorous derivation of Young's double slit formulae.

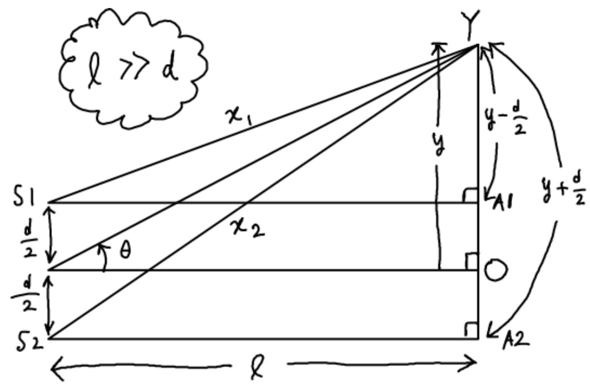
The diagram below summarizes the basic setup.



The two narrow slits S1 and S2 define two point sources of coherent light. Spherical waves of light emanate from these two points. When these two spherical waves interfere, they form various minima and maxima in intensity. This interference pattern can be observed by placing a screen as shown and measuring the intensity of light there.

Note that this problem is very similar to the sound interference problem involving two speakers.

In order to treat this problem mathematically, let us consider the following diagram.



An important assumption[1] is that $l \gg d$. Under this single assumption, the following conditions can be derived.

$$d \sin\theta = m\lambda \qquad \text{constructive interference} \qquad (16.1)$$

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad \text{destructive interference} \qquad (16.2)$$

$$m = 0, \pm 1, \pm 2, \ldots \qquad (16.3)$$

The bright light one gets due to the $m = 0$ constructive interference condition is called the zero-th order light. The bright light one gets due to the $m = \pm 1$ constructive interference condition is called the first order light. And similarly for the second order light and higher order lights.

Let us derive the above conditions from the elementary considerations of two spherical waves interfering. The following derivation is *not* required, but optional for you to learn. The wave amplitude[2] at y is given by

$$D(y,t) = D(x_1,t) + D(x_2,t) \qquad (16.4)$$

$$= \frac{A}{x_1}\sin(kx_1 - \omega t + \phi) + \frac{A}{x_2}\sin(kx_2 - \omega t + \phi) \qquad (16.5)$$

$$\approx \frac{A}{x_1}\sin(kx_1 - \omega t + \phi) + \frac{A}{x_2}\sin(kx_1 - \omega t + \phi + k\Delta x) \qquad (16.6)$$

[1]This type of assumption is routinely made in a "scattering experiment."

[2]This usage of the word "amplitude" is common and should not be confused with A as amplitude. One often refers to the whole solution to a wave equation, such as $D(y,t)$ here, as "amplitude."

where $\Delta x \equiv x_2 - x_1$. So far, our expressions have been very general, and apply even when l is very small compared with d. However, now, we apply our assumption $l \gg d$. If this assumption is made, it is convenient to define

$$r \equiv \sqrt{l^2 + y^2} \tag{16.7}$$

From the above diagram, $x_1 = \sqrt{l^2 + \left(y - \frac{d}{2}\right)^2}$. Clearly, $x_1 \approx r$. We can do better, by evaluating the first order term in d using the Taylor expansion theory of calculus. We can do similarly for x_2. We get

$$x_1 = r - \frac{yd}{2r} + O(d^2) \tag{16.8}$$

$$x_2 = r + \frac{yd}{2r} + O(d^2) \tag{16.9}$$

Plugging these results into the expression for $D(y,t)$, we get

$$D(y,t) \approx \frac{A}{r}\left(\sin X + \sin(X+\delta)\right) \tag{16.10}$$

$$X \equiv kx_1 - \omega t + \phi \qquad \text{common phase} \tag{16.11}$$

$$\delta \equiv k\Delta x = k(x_2 - x_1) \approx \frac{kyd}{r} = kd\sin\theta \qquad \text{phase difference} \tag{16.12}$$

Clearly, we get a constructive interference if the phase difference δ is 2π times an integer, and a destructive interference if δ is an odd integer times π, since $\sin(X+\pi) = -\sin X$. Thus, we get

$$k\Delta x = 2\pi m \approx kd\sin\theta \Leftrightarrow d\sin\theta = m\lambda \tag{16.13}$$

$$k\Delta x = 2\pi\left(m + \frac{1}{2}\right) \approx kd\sin\theta \Leftrightarrow d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \tag{16.14}$$

proving what we set out to prove.