Notes for Lecture 13

Lens and mirror equation

Please read the book and your lecture notes taken during classes to find out all we covered in Lectures 13 and 14. Here, we cover only some tricky/fundamental concepts.

For a spherical/plane optical element (mirror or thin lens), the following equation is valid

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_f} \tag{13.1}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad \text{lateral magnification} \tag{13.2}$$

under the usual assumption of small angles involved, or equivalently small object size compared to f.

One main thing to note is that all of the quantities above can be negative. If d or f is negative, then the associated thing (object, image, or focus) is called "virtual." If d or f is positive, then the associated thing is called "real."

As explained in LN 12, "virtual" and "real" can be understood as meaning "no relevant light" and "existing relevant light" around the thing under discussion. What does the "relevant light" mean? For an object, the relevant light is the incoming beam of light. For an image/focus, the relevant light is the transmitted light for lens, and the reflected light for mirror. This physics is quite easy to understand in a single optical element experiment. However, if more than one optical element is used, then this can lead to potential confusion. How to avoid confusion? This leads to the following general guidelines that must be heeded to carefully.

1. **Consider one optical element at a time!** When one draws a diagram for one optical element out of multiple elements used, the best practice is to forget

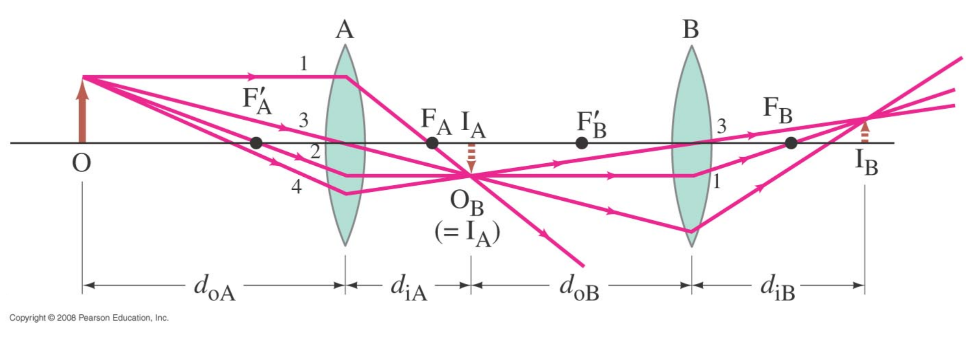
temporarily all other optical elements and how the real light behaves! This is because the real-ness or the virtual-ness of object and iamge is determined only within the context of the diagram for *one* optical element of interest at a given time.

2. **When two optical elements are used in succession, then the image of the first optical element becomes the object of the second element.** This "chain rule" makes it possible to put together information that is gained by considering one optical element at a time. Note that just because the first element creates a virtual image does not mean that it is a virtual object for the second element! This real-ness and the virtual-ness of the same thing can change, as **the real-ness or the virtual-ness is determined per diagram of a given optical element**.

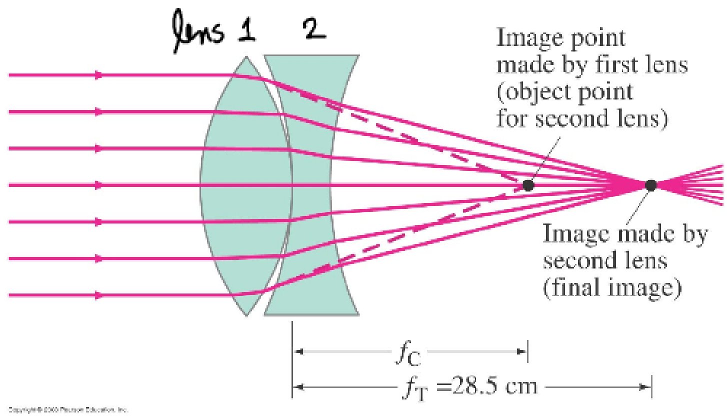
3. **Eq. 13.1 applies only to one optical element.** In particular, d and f are measured relative to the optical element in question. In other words, the zero of d or f is defined as the position of the optical element. So, in a multiple element problem, it is important to "shift the zero" of d and f properly as one shifts to a different optical element.

To appreciate what the above guideline really means, one must try to do actual problems, such as examples in the book or homework/practice-exam problems. Here, we consider a few two lens systems to illustrate our points.

Here is an elementary example, a two converging lens system, with $d_{oA} > f_A$ and $d_{oB} > f_B$. First, the first lens (A) forms a real image (since $d_{oA} > f_A$) at d_{iA}. Second, this real image becomes a real object for the second lens (B), which then maps it to another real image at d_{iB}.



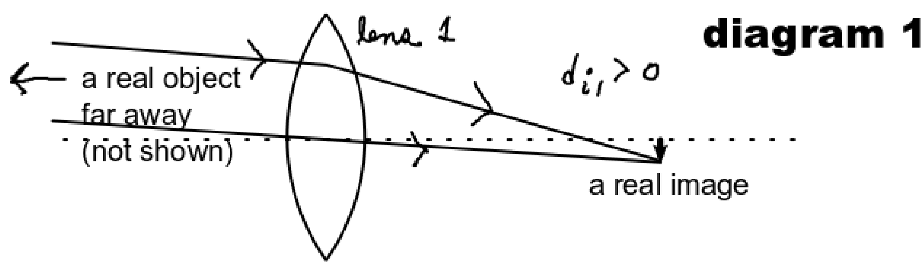
Now, let us consider a slightly more complicated situation, as shown below. It is important to focus our attention to diagrams 1 and 2, as those diagrams, and only they, not the full diagram shown at the top, are relevant to Eq. 13.1, and to determining the real-ness or the virtual-ness of things.

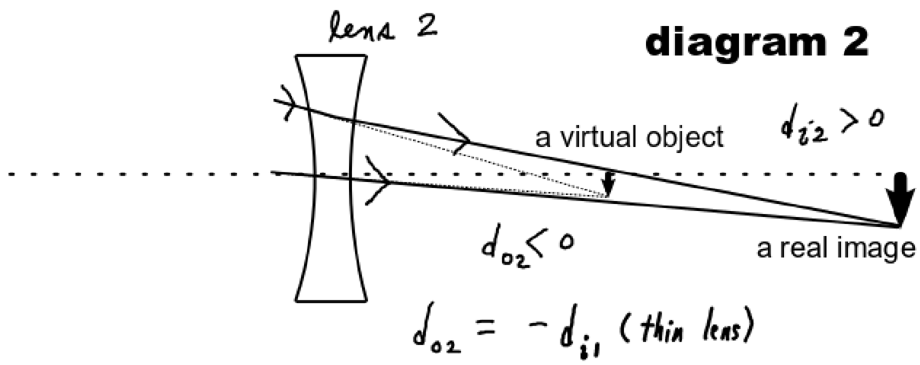


Twin lens system

a weak diverging lens followed by a strong converging lens

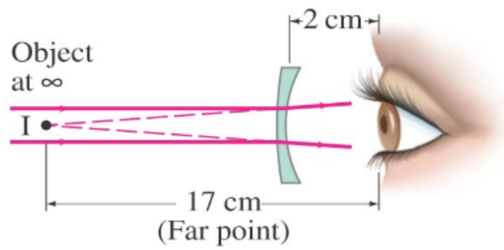
If this system is used to focus a distant object (not an infinitely distant object as depicted above), then we get the following.





Note that, *in diagram 1*, there *is* transmitted light around the image. So, the image is real. However, this image becomes the object for lens 2 (diagram 2). *In diagram 2*, there *is no* incoming beam of light around this object. So, we have a virtual object, and must use a negative value for d_{o2}. Assuming thin lenses, we get $d_{o2} = -d_{i1}$.

Let us consider a third example involving a corrective lens of a near-sighted eye. Again, the situation is broken down into two diagrams. In diagram 1, note that there is no transmitted light around the image. So, we got a virtual image. We must use $d_{i1} < 0$. However, this image acts as an object to the eye. From the eye digram (diagram 2), there *is* light around this object, and thus it acts as a real object. So, $d_{o2} > 0$.



This situation can be viewed as describing a two lens system, also. Lens 1 is the corrective lens shown. Lens 2 can be taken to be the lens of the eye (or more correctly, it must be taken as cornea + lens of the eye).

When this twin lens sytem is used to view a distant object (not at infinite distance), then the following diagrams apply to lens 1 and lens 2.

diagram 1 what lens 1 (corrective lens) sees

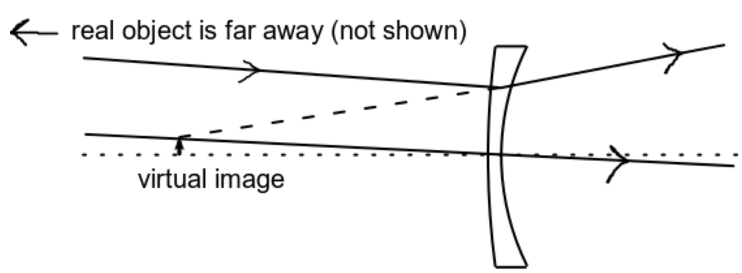
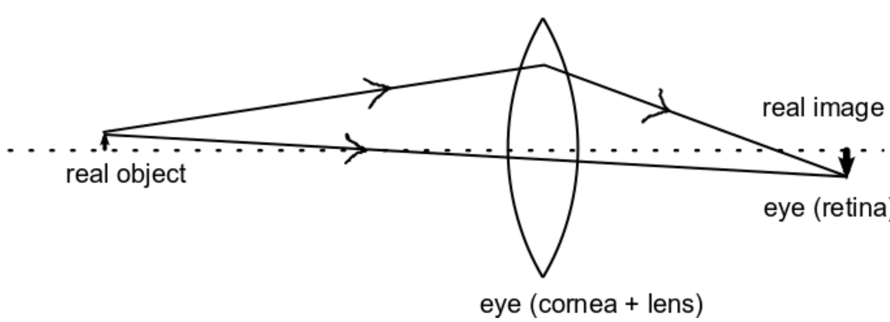
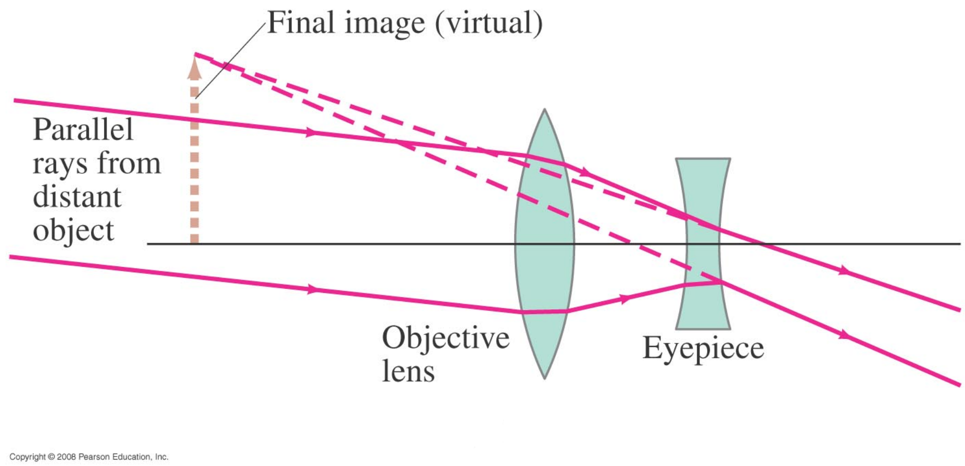


diagram 2 what lens 2 (eye) sees



Lastly, let us consider the following situation, the design for a Galilean terrestrial telescope or an opera glass.



Please answer these questions. Is the image of the objective lens real or virtual? Is the object of the eyepiece real or virtual? How about the image of the eyepiece?

To close, the following can be regarded as quick tricks, sometimes, for finding the real-ness and the virtual-ness of objects and images (including the focal point, which is the image of an infinitely distant real object).

1. If a thing is on the right side, then it is real. If on the wrong side, then it is virtual. The right side is defined as the same side where the relevant light is. The wrong side is defined as the opposite side to the right side. What divides these two sides? The optical element itself does so. So, an object is real if it exists on the side where the incoming beam is. An image that forms on the back of the mirror is virtual, since the relevant light, the reflected light, for the image is not there, but on the front side of the mirror. Virtual images of lenses exist on the wrong side where the incoming beam is, not on the right side where the transmitted beam is.

2. If the input beam is divergent on the optical element, then it means a real object. If the input beam is convergent on the optical element, then it means a virtual object. For an image, the opposite holds. If the output beam forms a converging beam, then the image is real. If the output beam forms a diverging beam, then the image is virtual.

Please use both of these quick tricks, to review the above four different situations depicted in four figures.