Notes for Lecture 12

Mirrors, Refraction, Dispersion

12.1 Virtual or real?

Generally, "virtual" means mathematically possible but not realized. We already had some discussion of it in the context of the mathematical view of the superposition principle (cf., reading quiz 4). For example, the superposition of two waves considered in Section 7.3 was the superposition of two *virtual* waves (the first example in page 6 of LN 7). Those two waves were mathematical objects, which do not exist as separate entities in the real world. If they did, then they should not break the energy conservation principle! In contrast, the two waves considered in the 2nd paragraph of page 6 of LN 7 are real.

The same definition applies here as well, actually. If we form an image using an optical element, or a combination of optical elements, then, there must be a diverging light wave emanating from that image. Such a diverging light wave is what makes us perceive the image. Why so? It is the same reason why each real object acts exactly the same way: we see things, since each thing reflects off ambient light and become a source of a diverging light wave[1]. For a real image, this diverging light wave can be traced all the way back to the image itself. I.e., there truly is light around a real image, and it does emanate from the image. In contrast, the light wave that appears to emanate from a virtual image *cannot* be traced all the way to the image itself. As far as this light wave is concerned, it does *not* exist at or very close to the virtual image. That light that does not actually exist is simply a mathematical possibility that is not yet realized; that light at the image is virtual.

An easy example of a virtual image is an image formed in a plane mirror. The

[1]This wave can be approximated as a spherical wave at large distance.

image forms behind the mirror, but we know that there is no such light behind the mirror (Figure T32-7)! Likewise, all images of a convex mirror are virtual (Figure T32-19; However, I recommend you to prove this statement, using the mirror equation, below!). Concave mirrors can produce both virtual (Figure T32-17) and real (Figure T32-16) images. You have seen an example of a real image made by a concave mirror: the little pink piggy appearing magically at the top of a "secret chamber" in the classroom demo (or, rather, the demo after class). Another example is the real image of the light bulb that appears "magically" in a youtube video linked from the forum web site.

It follows that the focal point of a convex mirror is always virtual, since the focal point is simply the image point of a very distant object. The light that appears to come from that focal point never exists around that focal point (Figure T32-19). In contrast, the focal point of a concave mirror is always real (Figure T32-14).

As we have been considering a single mirror so far, we did not consider the possibility of a virtual object at all, but a virtual object *can* occur if multiple optical elements are used together. As we shall see later, for a virtual object ($d_o < 0$), the light appears to *converge on* the object, rather than emerge from it!

12.2 Mirror equations and graphs

Here, couple of comments are made to remind you of what we did in the *last* class. The mirror equation $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$ can be rewritten as

$$d_i = f + \frac{f^2}{d_o - f} \tag{12.1}$$

Students of optics must be able to plot the *function* $d_i(d_o)$, i.e. d_i as a function of d_o, as a graph, when the range of d_o is given and the value of f is given. We did this, in class, for a convex mirror ($f < 0$) with a real object ($d_o > 0$), and found that d_i starts from 0 (for $d_o = 0$) and steadily decreases to f as $d_o \to \infty$. Also, we were able to prove that, for $f < 0$ and $d_o > 0$, $|m| < 1$: so a convex mirror always de-magnifies.

12.3 Index of refraction

Due to the interaction of light and matter (electrons, mostly, and also other particles such as protons), light slows down in matter. This is summarized by

$$n \geq 1 \qquad \text{index of refraction} \tag{12.2}$$

$$v = \frac{c}{n} \qquad \text{speed of light in a medium} \tag{12.3}$$

$$\lambda = \frac{\lambda_v}{n} \tag{12.4}$$

where in the last equation, λ is the wave length in medium, while λ_v is the wave length in vacuum. As usual, here we are concerned with v of a travelling wave, not a standing wave ($v = 0$). This last equation above ensures that the frequency of a given monochromatic light wave never changes (Section 8.1). Inside the medium, the frequency must be given by $f = v/\lambda$, and this matches the frequency in vacuum c/λ_v. Lastly, c is the speed of light in vacuum = 3.00×10^8 m/s, one of the most important constants of Nature.

12.4 Snell's law

This law can be derived in a variety of ways, using Fermat's principle (as done in the forum), energy-momentum consideration (as you can easily do *if* you know a bit of modern physics and make use of the above definition of the index of refraction; I am not requiring you to do this, yet, naturally), or the Huygens/Feynman view of how light propagates (each point of a wave front is itself a new source of wave, and all of those waves interfere!). However, in this course, it is more important to know how to *use* Snell's law, than to know how to derive it. The law is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{See Figure T32-21 for the defintion of symbols.} \tag{12.5}$$

12.5 Light through a slab

If we have a regular slab of material as shown in Figure T32-24, the net effect of the two refractions through the two flat parallel surfaces of such a slab, is the shifting of light. The light comes through the slab at the same angle θ_B, as it was incident, but, the light source would appear to be shifted with respect to the true source.

12.6 Apparent depth

If you dropped your wedding ring in a swimming pool and you estimate that that ring is at a certain depth d_a by observing it from outside the water (d_a = apparent depth), then you will have to realize that the true depth is given by $d = nd_a$ where n is the index of refraction of water. Likewise, objects in medium with high n (say n_h) look shorter than they really are, when they are observed from within a neighboring medium with low n (say n_l): the reduction ratio for the apparent vertical length (assuming that the interface between the media is a horizontal plane) is given by n_l/n_h. This has been derived in class; you can also read Example T32-9.

12.7 Dispersion

When light spectrum splits and colors appear separated from a medium such as prism or water drop (causing a rainbow), we say that the light is dispersed.

For physics-minded students like you, the word "dispersion" can be best defined to mean the following:

$$n = n(k) \qquad \text{index of refraction depends on wave number/length} \tag{12.6}$$

where $k = 2\pi/\lambda$ is the wave number inside the medium. Indeed, this is the reason why visible light would "disperse into different colors" inside a medium. However, the word dispersion is used much more generally, even when the spectrum outside the visible light is considered[2]. When the index of refraction is dependent on wave number, then, by Eq. 12.3, the speed of light is dependent on the wave length/number (and thus on frequency or color as well)!

We learned in Section 5.1 that for a travelling sinusoidal wave, the relation, $\omega = vk$, is always valid. In a dispersive medium, we have, for light,

$$\omega = v(k)k = \frac{c}{n(k)}k \tag{12.7}$$

For this reason, the functional $\omega(k)$ is *always* called the **dispersion relation**, even if $\omega(k)$ is linear in k. However, note that the medium is dispersive, only when $\omega(k)$ is a non-linear function of k. Read more about the dispersion relation in previous lecture notes: page 4 of LN 5 and Section 7.1.

While the dispersion is responsible for beautiful rainbows (see next), it is also responsible for the signal degradation when light pulses are sent in optical cables (as

[2] And indeed for any wave, not just light.

for the internet communication!). This can be understood easily since the information contained in light pulse trains must necessarily use non-monochromatic light. Any small dispersion will have a grave consequence after a long distance of light travel, since each light pulse will broaden and become less tall, due to the dispersion. Initially, all different Fourier components of light go the same distance, but after a long distance of travel, different speeds for different wave lengths of light will lead to a broadening of the pulse, just as a group of runners become wide spread in space and time after a while in an Olympic running competition. If any two adjacent light pulses become merged due to such pulse degradation, then it means a true loss of information. The original pulse train must be regenerated, i.e., the pulses must be sharpened up again, before this happens.

12.8 Rainbow

Please do the last problem at the end of Chapter T32, if you want to know more about the Rainbow scattering. The rainbow results from dispersed light spectrum appearing at scattering angle $\phi \approx 139$ degrees (Figure T32-67) due to the two refractions and one internal reflection that light goes through in each rain drop. A significant fraction of light scattering is concentrated near this angle. Often a double rainbow can be observed. The second bow in a double rainbow results from the light that goes through two internal reflections, instead of one; this light starts from the *lower* part (as opposed to the upper part for the primary rainbow) of a raindrop and emerges at an angle that corresponds to scattering angle $\phi \approx 129$ degrees. For this reason, the second bow appears higher, and its color ordering is the opposite of that of the primary rainbow.

12.9 Total internal reflection

Consider the case $n_1 > n_2$. Snell's law, Eq. 12.5, definitely cannot be satisfied if $\theta_1 = \pi/2$, since the maximum of the sine function is 1! In fact, one can see that $\frac{n_1}{n_2}\sin\theta_1$ must not be greater than 1. This defines a critical angle, θ_c,

$$\theta_c = \sin^{-1}\frac{n_1}{n_2} \tag{12.8}$$

$$\text{total internal reflection if } \theta_1 > \theta_c \tag{12.9}$$

Note that $\sin^{-1}$ is the inverse sine function (sometimes referred to as arcsin), and is *not* 1 over sine. Please read more about the critical angle in Section T32-7.